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NASTRAN THERMAL ANALYZER - THEORY AND APPLICATION INCLUDING A GUIDE TO MODELING ENGINEERING PROBLEMS

Volume I

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16. Abstract

The NASTRAN Thermal Analyzer ($\tilde{N}$TA) is a new and unique general-purpose heat transfer computer program based on the finite element method. It is an integrated part of the NASTRAN system. To fill the needs of current and potential $\tilde{N}$TA users who, in general, would neither be familiar with the underlying finite element theory nor have any prior experience in NASTRAN modeling, two volumes have been written to provide thermal engineers with a comprehensive and self-contained manual encompassing theory, application, and examples of modeling.

This volume, The NASTRAN Thermal Analyzer Manual, is devoted to educating the $\tilde{N}$TA users with the fundamental and the theoretical treatment of the finite element method with emphasis on the derivations of the constituent matrices of different elements and solution algorithms. Necessary information and data relating to the practical applications of engineering modeling are included.

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FOREWORD

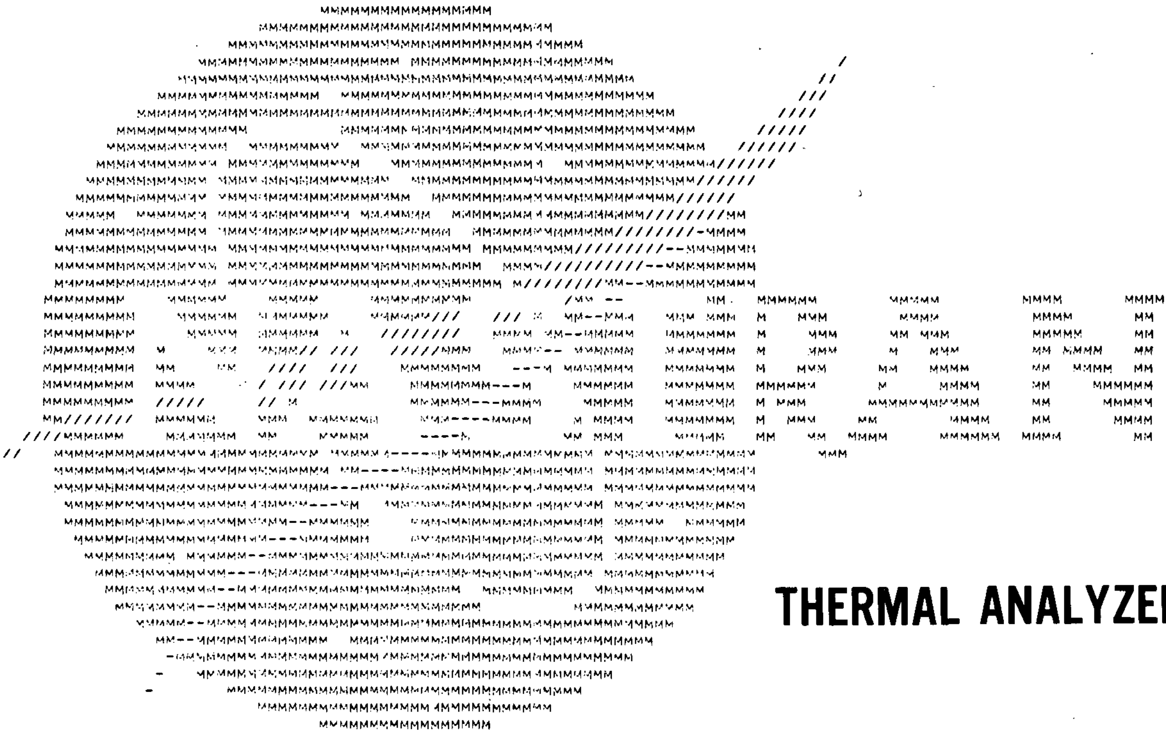
Without the guidance of a well prepared instruction and reference manual, a new user of the NASTRAN Thermal Analyzer ($\bar{\text{N}}$TA) may have difficulty in proceeding. Unless the user is a member of a large team where at least one member is familiar with the finite-element method and the NASTRAN modeling, he will have to search through widely scattered literature and documents before finding what he needs. Recognizing this problem, we have utilized the knowledge and experience acquired during the development and application of the $\bar{\text{N}}$TA to provide essential and sufficient information required by those who use it.

With both beginners and experienced thermal analysts in mind, the NASTRAN Thermal Analyzer Manual has been carefully designed to ensure a useful, comprehensive, and self-contained document that encompasses the underlying theories, instructions of modeling construction, descriptions of various data formats, and examples of engineering modeling. The emphasis throughout has been placed on the practical aspects relevant to the use of the $\bar{\text{N}}$TA.

The first edition was published as two NASA/GSFC X-documents (X-322-76-16 and X-322-76-17) in December 1975. Its contents evolved from the lecture notes used during the in-house training course conducted in mid-May 1975. Source materials of this first volume consisted of expanded extractions of published and unpublished works resulting from research and development efforts in the development of the $\bar{\text{N}}$TA, together with other pertinent materials scattered in the three voluminous NASTRAN manuals of the Level 15.5 version.

The phenomenal number of requests for the two publications from both domestic and international users reflects the general enthusiasm and the rapidly expanding use of the $\bar{\text{N}}$TA. It is evident that the $\bar{\text{N}}$TA is increasingly important for many space, military, and industrial applications, particularly in unified thermo-structural analyses. To benefit $\bar{\text{N}}$TA users, these two volumes are to be published as formal NASA publications to ensure a wider distribution and lasting availability.

H. P. Lee



THERMAL ANALYZER

CONTENTS

		Page
ABSTRACT		i
FOREWORD		iii
1.	INTRODUCTION	1
1.1	A Brief History of the $\overline{N}$TA Development	9
1.2	Distinctions Between the Finite Difference Method and the Finite Element Method	15
1.3	The NASTRAN Thermal Analyzer Capabilities	17
2.	FINITE ELEMENT THEORY IN HEAT TRANSFER APPLICATIONS	19
2.1	Variational Principle	20
2.2	A One-Dimensional Steady-State Conducting Rod	23
2.2.1	*Finite Element Formulation*	23
2.2.2	*Finite Element Formulation in Matrix Notation*	30
2.3	Finite Element Formulation of a Two-Dimensional Triangular Element	38
2.3.1	*Conduction with Linear Boundary Conditions*	38
2.3.2	*Nonlinear Radiative Boundary Condition*	42
2.4	An Analogy Between Structural and Thermal Systems	46
2.5	Constituent Matrices for Various Elements	49
2.5.1	*Heat Conduction Elements*	49
2.5.2	*Boundary Surface Elements*	56
2.6	Methods of Solution	61

CONTENTS (Continued)

		Page
2.6.1	*Linear Steady-State Thermal Analysis*	61
2.6.2	*Nonlinear Steady-State Thermal Analysis*	64
2.6.3	*Transient Thermal Analysis*	71
2.7	**Rigid Format DMAP Listings**	77
2.7.1	*DMAP Sequence for Linear Steady-State Thermal Analysis*	78
2.7.2	*DMAP Sequence for Nonlinear Steady-State Thermal Analysis*	84
2.7.3	*DMAP Sequence for Transient Thermal Analysis*	91
3.	**THE NASTRAN THERMAL ANALYZER INPUT DATA DECK**	103
3.1	**Overview of Thermal Model Preparation**	103
3.2	**General Description of Data Deck**	109
3.3	**Executive Control Deck**	112
3.3.1	*Executive Control Card Descriptions*	112
3.3.2	*Executive Control Deck Examples*	116
3.4	**Case Control Deck**	118
3.4.1	*Subcase Definition*	118
3.4.2	*Data Selection*	119
3.4.3	*Output Selection*	120
3.4.4	*Case Control Card Descriptions*	121
3.5	**Bulk Data Deck**	138
3.5.1	*Functional Description of Bulk Data Cards*	138
3.5.2	*Format of Bulk Data Cards*	149
3.5.3	*Bulk Data Card Descriptions*	151
REFERENCES		245

ILLUSTRATIONS

Figure		Page
1.1	Representative heat conduction elements	5
1.2	The telescope in the OSO-I Colorado experiment	11
1.3	Conduction elements in the finite element thermal model of the telescope in the OSO-I Colorado experiment	12
1.4	Boundary surface elements for radiative exchanges between the telescope and the casing walls of the OSO-I Colorado experiment	13
1.5	The IUE telescope thermal model	14
1.6	Distinction between the finite difference lumped-nodal method and the finite element method	15
2.1	True solution y(x) and trial function $\tilde{y}(x,\epsilon)$	21
2.2	Physical representation of a one-dimensional rod	24
2.3	Finite element representation of a one-dimensional rod	24
2.4	A 2-D triangular element	39
2.5	Representative heat conduction elements	50
2.6	Definition of a 1-D rod element and a 2-D triangular element	53
2.7	Formation of a 2-D quadrilateral element	55
2.8	An ELCYL type (elliptic cylinder) of the HBDY element	60
2.9	Simplified flow diagram for linear steady-state thermal analysis	62
2.10	Simplified flow diagram for nonlinear steady-state thermal analysis	65
2.11	Simplified flow diagram for transient thermal analysis	72
3.1	Thermal model functional diagram	104
3.2	A complete NASTRAN THERMAL ANALYZER input deck	110
3.3	Grid point sequencing for one-dimensional systems	141
3.4	Grid point sequencing for surfaces	141
3.5	Sample NASTRAN data input sheet	152
3.6	HBDY element orientation (for QVECT flux)	165

TABLES

Table		Page
1.1	Cross-reference of the $\overline{\text{N}}$TA sample problems vs. thermal analysis features demonstrated	8
1.2	Number of model required for thermo-structural analysis including radiative exchanges	16

CONTENTS OF VOLUME II

		Page
ABSTRACT		i
I.	INTRODUCTION	1
II.	EXPLICATION OF THE $\bar{N}$TA SAMPLE PROBLEMS	2
A.	Physical Description of the Basic Problem	2
B.	Generation of the Linear Steady-State (LSS) Finite-Element Model	2
	1. *GRID Points*	2
	2. *Heat Conduction Elements*	3
	3. *Boundary Surface Elements*	6
	4. *Constraints*	6
	5. *Loads*	8
C.	Discussion of the Sample Problems	8
	1. *Sample Problem 1*	12
	2. *Sample Problem 2*	19
	3. *Sample Problem 3*	25
	4. *Sample Problem 4*	28
	5. *Sample Problem 5*	29
	6. *Sample Problem 6*	30
	7. *Sample Problem 7*	32
	8. *Sample Problem 8*	32
	9. *Sample Problem 9*	32
	10. *Sample Problem 10*	33
	11. *Sample Problem 11*	35
	12. *Sample Problem 12*	36
	13. *Sample Problem 13*	37
	14. *Sample Problem 14*	37
	15. *Sample Problem 15*	39
	16. *Sample Problem 16*	39
	17. *Sample Problem 17*	41
	18. *Sample Problem 18*	42

CONTENTS OF VOLUME II (Continued)

Page

19. Sample Problem 19 43
20. Sample Problem 20 44

III. REFERENCES 45

APPENDIXES

A CROSS-REFERENCE OF THE N̄TA SAMPLE PROBLEMS VS. THERMAL ANALYSIS FEATURES DEMONSTRATED A-1

B INFORMATION AND WARNING MESSAGES PRESENT IN THE N̄TA SAMPLE PROBLEMS B-1

C COMMON N̄TA USER ERRORS C-1

D HOW TO DOCUMENT A NASTRAN ERROR D-1

E DANGERS IN THE USE OF NASTRAN NONLINEAR LOADS IN STRUCTURAL TRANSIENT ANALYSES E-1

F N̄TA SAMPLE PROBLEM LIBRARY F-1

NASTRAN THERMAL ANALYZER

THEORY AND APPLICATION INCLUDING A GUIDE TO MODELING ENGINEERING PROBLEMS

Hwa-Ping Lee

1. INTRODUCTION

The unique finite-element based NASTRAN Thermal Analyzer ($\overline{\text{N}}$TA)[1] * is a general purpose heat transfer computer program. As an integrated part in the NASTRAN (NAsa STRuctural ANalysis computer program) system,[2] this thermal analysis capability is fully capable of rendering temperature solutions and heat flows in solids subject to various boundary conditions which range from prescribed temperatures at grid points and specified thermal loads to convective and radiative modes of heat transfer at boundary surfaces in both steady-state and transient cases. This heat transfer computer program has been developed by an application of existing functional modules in the NASTRAN, which were designed originally for the purpose of structural analysis, and the addition of new modules including new elements and new solution algorithms. In addition to being an independent heat transfer computer program, $\overline{\text{N}}$TA has the unique feature of being completely compatible with the structural counterpart in the NASTRAN system with regard to both capacity and the finite-element model representation. Therefore, the $\overline{\text{N}}$TA as a part of the unified thermo-structural analysis capabilities in NASTRAN is especially suited to compute temperature results for thermally sensitive structural problems which may have very large sized and complex configurations.

This heat transfer analysis capability has been integrated into the NASTRAN system in its Level 15.5 version, which has been available for general use through the NASA software dissemination apparatus, Computer Software Management and Information Center (COSMIC), since June, 1973.

The original $\overline{\text{N}}$TA documents were presented as a supplement to the three NASTRAN manuals, namely, the NASTRAN Theoretical Manual,[3] the NASTRAN User's Manual,[4] and the NASTRAN Programmer's Manual.[5] Such an arrangement in three voluminous structurally oriented documents did not readily lend itself to easy access but discouragement to potential users. Since the $\overline{\text{N}}$TA has made maximum utilization of the applicable functional modules available in NASTRAN for economic reasons, a number of structurally oriented input cards are directly useable in thermal applications. Many terms used in the titling and mnemonics either in input cards or in output display were poorly interpreted or inadequately explained. These facts together with the unfamiliarity with the finite element method would undoubtedly dissuade many potential users away from learning to use it. Although mathematically sophisticated, the $\overline{\text{N}}$TA is, in fact, very flexible, versatile and easy to learn insofar as how to prepare all the input data and control options given a well prepared instruction and reference manual.

*Superscripts denote references at end of this volume.

The objective of this publication is to provide a comprehensive and self-contained manual encompassing theory and application to thermal analysts who, in general, would neither be familiar with the underlying finite-element theory nor have any prior experience in NASTRAN modeling. Physically meaningful heat transfer terms known to general thermal analysts have been used throughout. This two-volume set manual entitled "NASTRAN THERMAL ANALYZER–Theory and Application Including a Guide to Modeling Engineering Problems" has been designed for self-study and easy reference.

The contents of this Volume I, the NASTRAN Thermal Analyzer Manual, is divided into three chapters. A brief account of developmental history of the $\overline{\text{N}}$TA computer program, major distinctions of the finite element method versus the finite difference method in thermal applications and a summary of the $\overline{\text{N}}$TA capabilities are given in three separate sections, 1.1 through 1.3 later in this chapter.

The uniqueness of the $\overline{\text{N}}$TA computer program lies wholly in its underlying theoretical basis, the finite element method. The use of this method has grown from a limited structural application by Turner et al.,[6] as recently as 1955 to one of the most active fields in the numerical analysis of problems related to mathematical physics. Theoretical developments and engineering applications flourishing in the literature were largely in the structural discipline. The exploratory works applying the finite element method to thermal field were rather restrictive, and researchers who investigated the heat conduction problems, e.g., Zienkiewicz[7] and Visser,[8] had a structural background. It was attributed to the fact that mathematical analogy did exist between the two distinct structural and thermal disciplines. The interest in solving thermo-stress problems prompted the use of available structural computer programs to solve temperature problems whose results were required to compute thermally induced stresses or deflections. As a consequence, a systemmatically presented literature dealing with theoretical aspect of the finite element method using thermally oriented terms has been scarce. An understanding of the basic finite element theoretical relationships would not only ensure modeling correctness and effectiveness but enable the user to extend the $\overline{\text{N}}$TA capabilities to special applications which are rather not apparent at the first glance. In addition, it would be also helpful in assessing accuracy afforded by the numerical solution.

Theoretical developments in finite element analysis have placed great dependence on the calculus of variations. The finite element method contrasts with the conventional finite difference method in that the latter is a numerical process applying a direct approximation approach to the governing differential equation while the former is an approximation applied to the variational terms. A thorough understanding of the calculus of variations is not necessary to use the finite element method, however, an introduction will be valuable because the finite element formulation for element matrices starts with a much different form than the usual method to which users have been exposed in academic training or engineering practice. The variational principle serves as an essential link to bridge what one has been familiar with and the necessary conversions for the finite element formulation.

In heat transfer applications, the discretization in finite element manner can be achieved via several methods within the confines of the variational principle. The most widely accepted approach is a process referred to as the Rayleigh–Ritz procedure[9,10] with less restrictions to the complexities inherent in its conventional application. Following this procedure, the problem is to find a temperature function which minimizes a specific functional (dependent on unknown functions appearing in the form as an integral) over the entire region. If this unknown temperature function is uniquely specified throughout the region by a discrete number of its values (expressed in terms of coordinates of the grid points which define the subregions or "finite elements") and the value of the temperature functions of a particular grid point influences only the temperature function in the adjacent elements, then the minimization of the functional throughout the whole region with respect to grid point temperature unknowns (which are treated as parameters in the Rayleigh-Ritz method) results in a set of simultaneous equations. The solution of these equations yields an approximate solution to the original problem. With this procedure in mind, one may illustrate the detailed formulation.

While approximate minimization of a functional is the most widely accepted procedure of arriving at a finite element formulation, it is by no means the sole approach possible. Identical formulation can be achieved alternatively by the use of the Galerkin weighted residual method[9,10] which, in fact, is even simpler to apply than the Rayleigh-Ritz method. It should be noted that with a given variational formulation, there is an equivalent derivation based on the Galerkin procedure, but the converse does not always hold because the Galerkin method is independent of the existence of a variational principle. Zienkiewicz and Parekh[11] demonstrated this approach to obtain 2-D and 3-D isoparametric finite elements. No analytical treatment of the Galerkin weighted residual method will be included in this manual.

Another alternative approach is to draw the analogy of thermal potential energy from the principle of minimum potential energy of structural analysis as demonstrated by Visser.[8] Expressions for a thermal system can be obtained readily with variables and parameters interpreted appropriately from the structural system. Obviously, this approach is more direct than any one method described previously but it is not suitable for one without a structural background. This approach, therefore, will not be elaborated upon until a later section after the reader has become experienced in the concept and mathematical manipulation of the basic finite element theory.

The basic and general treatment of the finite element method for element formulation is considered in the three beginning sections. Section 2.1 provides the basis of the transition from a variational statement of a problem to an equivalent governing differential equation and establishes the fact that the solution satisfying the Euler–Lagrange equation fulfills the necessary condition for the functional to be stationary. Consequently, the variational formulation of a physical problem is readily obtained by reversing the steps to treat the governing differential equation as the Euler–Lagrange equation associated with the specified boundary conditions. The theoretical foundation is then laid for subsequent finite element formulations.

To illustrate the equivalence of the variational and boundary value problems and the formation of a set of algebraic equations derived from the finite element method, a simple steady-state case of a 1-D conducting rod having a uniform internal heat source in a convective environment with prescribed end boundary conditions has been selected as a demonstration problem in section 2.2. The 1-D case is employed extensively in examples of basic theory including the element formulation and an assembly of the system equations. For clarity and in order to provide a step-by-step comparison, equations are expressed in algebraic form first, then the derivations are repeated using the matrix notation. The matrix representation is the standard form associated with the finite element formulation with its benefit being its compactness and suitability for computer application.

The element formulation is then extended to a 2-D triangular element in section 2.3. The transient case with general thermal boundary conditions including prescribed temperatures at grid points, specified heat flux, convective heat transfer, and radiative exchanges is taken into consideration. The derivations of element matrices for thermal conductance, heat capacitance, and thermal loads are given first, and a detailed treatment of radiative effects for diffuse-grey surfaces follows.

The $\overline{\text{N}}$TA computer program is a temperature solving capability added to the NASTRAN system. For economic reasons, functional modules available for direct use or needed for minor modifications were utilized to the maximum extent. The feasibility of using those functional modules originally designed for structural analysis in thermal applications lies in the existence of a mathematical analogy. Section 2.4 makes a comparison of the vibration equation with the heat equation when both are cast in finite element matrix form. The equivalences and conditions of being mathematically equivalent are discussed.

Individual element matrices for all thermal elements available in the element library of the $\overline{\text{N}}$TA computer program, such as shown in figure 1.1, are still needed as the basis for implementing new codes or modifying existing elements. Even when a mathematical analogy does exist between the structural and thermal systems, the analogy refers to the assembled global equation level. On the element level, each component in every matrix must be determined according to the physical problem being studied. It would be impractical to include element formulations for all elements as is done for the 1-D rod element and the 2-D triangular element in sections 2.2 and 2.3, respectively. However, a summary containing essential formulating steps and the resulting expressions for all basic elements available in the $\overline{\text{N}}$TA computer program is given in section 2.5. The contents are presented in an arrangement conforming to functional objectives of the computer program. In order to expose the user to a different approach of deriving the thermal conduction matrix, the approach via the thermal potential function[8] is now presented. As a part of the thermal conduction element to be used in transient thermal analysis, heat capacitance matrices in the consistent and lumped forms[12] are given. (The $\overline{\text{N}}$TA does use the lumped heat capacitance matrices for the consideration of efficient computer operations.)

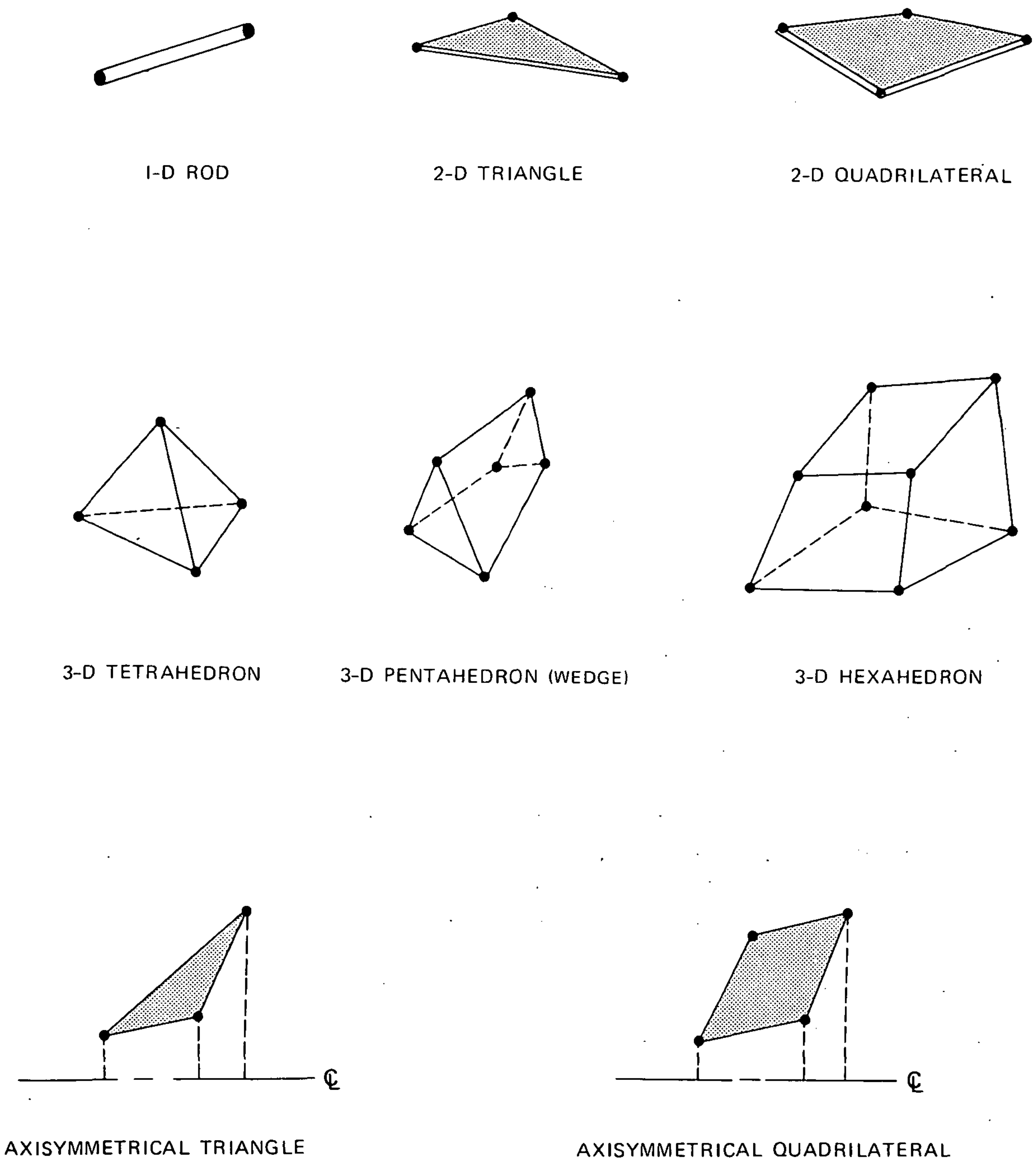


Figure 1.1. Representative heat conduction elements.

A special boundary surface element is provided to facilitate external thermal loads. This type of element is used to accept input thermal fluxes and to distribute equally the calculated total energy to the connected grid points. These grid points usually also define the thermal conduction elements. Six types of boundary surface elements are provided in this computer program. They are: (1) POINT–a flat disc, (2) LINE–a rectangular surface, (3) REV– a conical frustrum, (4) AREA3–a triangular surface, (5) AREA4–a quadrilateral surface and (6) ELCYL–an elliptic cylinder to provide numerous projecting surface areas by specifying two radii of the elliptic cylinder.

All types of the boundary surface element except the "ELCYL" can accommodate the following four types of external thermal loads: (1) Prescribed heat flux, (2) Convective heat flow, (3) Directional radiant flux from a distant source, and (4) Radiative exchanges between diffuse-grey surfaces. The "ELCYL" type is limited to the third type of external thermal load. Expressions to compute individual thermal loads are given in this section.

Two essential steps are involved in the solution of a physical problem by the finite element method. The first part is the element formulation (as has been discussed in preceding sections), and the second part is the method of solution. The solution algorithms available in the N̄TA computer program were designed so as to provide an accurate, efficient, and stable solution for a specific type of problem. For instance, the solution of a steady-state heat transfer problem can always be obtained via a transient solution route if a sufficiently large number of time steps are used in order to reach the steady-state result. Obviously, such an approach sacrifices the efficiency or economy of the computer operation. In the case of a nonlinear boundary value problem, with the concern of stability arising in the numerical solution, one has an added factor to be considered. The N̄TA has been provided with three specialized algorithms in the computer program to solve three types of problems classified as: (1) Linear steady-state problems–the solution is, in essence, a matrix inversion process, (2) Nonlinear steady-state problems–the solution is an iterative process essentially based on the Newton-Raphson method,[13] and (3) Transient heat transfer problems including both linear and nonlinear boundary conditions–the integration algorithm is a special form of the Newmark β method.[14] The N̄TA computer program inherent in the program structure of the NASTRAN consists of a number of mathematical modules (subprograms) that are executed according to a sequence of macro-instructions. Such a permanently stored prearranged sequence for solving a specific type of problem is called a "Rigid Format." Features and functional blocks of each rigid format together with associated flow diagrams are described in section 2.6.

To allow the experienced user to solve problems using features not accounted for in any one of the three rigid formats, the N̄TA permits the creation of a special program by adding and editing the program stored matrix routines called DMAP (Direct Matrix Abstraction Program). This is a user-oriented programming language of macro-instructions that must follow a set of programming rules in order to be interpretable by the N̄TA. A similar modification, but to a lesser degree, to alter a rigid format for one's own need is called ALTER. The listings of DMAP of the three rigid formats are included in section 2.7. This concludes the text dealing with basic theoretical considerations.

Chapter 3 furnishes information and data concerning the practical utilization of the N̄TA computer program. An overview of the thermal model preparation describing the functional relationships of coordinates, grid points, different types of elements, constraints, static and dynamic thermal loads, and specialized distinct rigid formats is outlined in section 3.1. The proper arrangement of required N̄TA input data cards into a data deck ready for the computer run submission is described in section 3.2. Functions and formats of individual cards to form three main parts of an N̄TA Data Deck, namely the Executive Control, Case Control and Bulk Data Decks, are explained in great detail in sections 3.3 through 3.5. New interpretation and explanation for titles and symbols appearing in all input cards, as necessary, are given. Terms irrelevant to the N̄TA modeling in cards belonging to the Case Control Deck are deleted from original cards or rephrased to reflect changes made for the N̄TA. To preserve the appearance of those dual-purpose Bulk Data Cards, symbols pertinent to structural analysis only but meaningless in thermal application are noted individually.

The second volume[15] is devoted to guiding the N̄TA users through examples. A sample problem library containing twenty problems covers all facets of the N̄TA modeling, and includes the selection of rigid formats, modeling techniques, control options and output interpretations. These sample problems have been carefully designed so as to use the same basic sample problem showing the steps needed for modeling specific features of the first problem and then being modified in succeeding problems by adding and/or replacing certain input data cards to demonstrate commonly used capabilities of the N̄TA. Comments are given regarding cards in the input data deck on a card-by-card basis. Additional explanatory statements are given in the text to clarify functional relationships between different types of cards and the interpretation of the results of a solution. They are given on a page-by-page basis in reference to the reproduced computer printouts. A summary cross-referencing the twenty examples in the sample problem library with features demonstrated in the N̄TA modeling is given in table 1.1 for an easy location of any specific modeling technique that can be imitated by beginners.

Table 1.1
Cross-reference of the $\bar{N}$TA sample problems vs. thermal analysis features demonstrated.

Thermal Analysis Feature	Sample Problem Number																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Linear Steady-State Run	✓												✓							
Nonlinear (radiation) Steady-State Run		✓				✓	✓	✓								✓	✓			
Nonlinear (radiation) Transient Run			✓	✓	✓				✓	✓	✓			✓	✓			✓	✓	✓
DMAP Alter(s)				✓	✓					✓	✓		✓					✓		
Structure Plot					✓															
Thermal Conductivity as F(T)						✓														
Convective Film Coeff. as F(T)							✓													
Anisotropic Thermal Conductivity as F(T)								✓												
Generate a Restart Tape and a Checkpoint Deck									✓											
Transient Printer Plots			✓	✓							✓									
Reduce Transient Printout Frequency				✓																
Define and Use a Set of GRID Points for Output			✓	✓					✓	✓	✓			✓	✓			✓		✓
Only SORT1 Transient Output				✓																
Produces Punched Output									✓		✓	✓								
Execute a Modified Restart										✓										
Produce Punched TEMP Cards During a Transient Run											✓									
Mixed SORT1 and SORT2 Transient Output											✓									
Cyclical Transient Loads											✓									
Automatically Generate RADMTX & RADLST Cards using the VIEW Program												✓								
Generate CHBDY Card Plots using a MacNeal-Schwendler $\bar{N}$TA Version													✓							
Uses SPC Card(s)	✓																			
Uses SPC1 Card(s)		✓				✓	✓	✓								✓	✓			
Transient Run Thermal Constraints			✓	✓					✓	✓	✓			✓	✓			✓	✓	✓
Uses MPC Card(s)	✓	✓	✓	✓		✓	✓	✓	✓	✓	✓			✓	✓	✓	✓	✓		✓
Multilayer Insulation															✓	✓	✓			
Effect of a Modified Guess Vector																	✓			
Gradient & Heat Flow Output																	✓	✓		
Printout of Thermal Mass Matrix																		✓		
$\bar{N}$TA Finite Difference Modeling																			✓	
Demonstrates Nonlinear Loads and Transfer Functions																				✓
Demonstrates OTIME Option																		✓		

1.1 A Brief History of the $\overline{\text{N}}$TA Development

The NASTRAN Thermal Analyzer was originated and developed at the Goddard Space Flight Center as one of the software products resulting from a research and development program entitled the Structural-Thermal-Optical Program (STOP).[16] The objective of this program was to provide analytical analysis capabilities in the multiple disciplinary areas with special attention to the interface problems interfering with reliable predictions of the thermo-stresses or deflections required to predict the optical performance of a large space telescope system exposed to changing orbital thermal conditions. The $\overline{\text{N}}$TA was specifically designed to permit efficient and precise interfacing between a thermal and a structural model, assuming that NASTRAN would be relied upon for structural analysis.

For a reliable thermo-structural solution, the thermoelastically uncoupled analysis requires accurate temperature inputs to the NASTRAN structural model. Prior to the existence of the $\overline{\text{N}}$TA, general purpose heat transfer computer programs were all of the lumped-nodal thermal network type (e.g., reference 17) and were based on the finite difference method. They were not only limited in capacity but seriously handicapped by incompatibilities arising from the model representations inherent in the two distinct approaches. The intermodel transfer of temperature data was found to necessitate extensive interpolation and extrapolation. This extra work proved not only a tedious and time-consuming process but also resulted in compromised solution accuracy. The R&D program STOP, therefore, undertook the development of a general purpose finite element heat transfer computer program to eliminate the need to form two independent models with the concomitant requirement for intermodel interpolation of temperature data. Studies were then conducted at GSFC aimed at achieving, efficiently and economically, a thermal analysis computer program that would ultimately be an integrated part of the NASTRAN system.

When this task began in late 1969, the theoretical aspects concerning the application of the finite element method to heat transfer analyses had been laid by Zienkiewicz and Cheung,[7] Visser,[8] and Wilson and Nickell.[18] The latter applied Gurtin's approach,[19] which deduced variational principles and explicitly incorporated the initial condition for linear initial value problems to linear transient thermal conduction problems. Efforts to broaden the scope of applications ranged from an extension to an axisymmetric problem by Brocci,[20] a study of the temperature-dependent thermal conductivity for an infinite slab by Aguirre-Ramirez and Oden,[21] a demonstration of inhomogeneous materials using higher order elements for a transient temperature analysis by Rybicki and Hopper,[22] and an illustration of a structurally well known condensation procedure to achieve efficient transient solution processes by the reduction of the order of the set of matrix differential equations by Gallagher and Mallett,[23] to many specific applications.[24-27] In addition, Emery and Carson[28] made a comprehensive evaluation of the accuracy and efficiency of finite element versus finite difference methods. Lemmon and Heaton[29] also compared the accuracy, stability and oscillation characteristics between these two numerical methods on one-dimensional problems. All studies however, were confined to conduction with linear boundary conditions. Only Richardson and Shum[30] included a simple blackbody radiative dissipation as a boundary condition in two transient

thermal examples. Since radiation is a major heat transfer mode in space-oriented applications and its presence introduces a fourth-power nonlinear temperature term that invalidates the known solution algorithms for linear heat conduction problems, the in-house studies were directed principally at investigating the effects of nonlinear thermal radiation on solution methods, accuracy and efficiency together with element behaviors of various heat conduction elements in combined modes of heat transfer analysis.

A detailed treatment of two basic heat elements including general thermal boundary conditions was presented by Lee.[31] Emphasis was placed at the formulation of constituent matrices with elaboration on the radiation matrix for the diffuse-grey surfaces and the appropriate solution methods for steady-state and transient thermal problems. This study provided not only an insight into the element behaviors but inspired the development of two separate solution algorithms for solving nonlinear steady-state and nonlinear transient problems in the $\overline{\text{N}}$TA efficiently and economically. A prototype computer program using the direct energy distribution method for the transient case was developed and coded by Heuser.[32]

Using NASTRAN (the structural version) to solve heat transfer problems directly by mathematical analogy and structural elements was accomplished by Mason.[33] The problems with simple radiatively dissipating boundaries had to be solved by a transient integration solution with many other restrictions as to the initial conditions and the depressed mass matrix originally associated with the acceleration term in the structural matrix differential equation. The radiative flux was simulated as a nonlinear load using nonlinear elements available in NASTRAN. This approach was later incorporated into the $\overline{\text{N}}$TA in the transient solution algorithm for the nonlinear radiative boundary.

While the NASTRAN Systems Management Office (NSMO) at Langley Research Center planned the extension of the NASTRAN capabilities to include linear thermal conduction analysis, the GSFC STOP program was seeking the implementation of a full-fledged finite element heat transfer computer program. The software capability was finally implemented by the MacNeal-Schwendler Corporation. It must be stressed, however, that a cooperative financial and technical effort between these two NASA centers made possible the emergence of this vital new capability in the NASTRAN system.

The public announcement of the $\overline{\text{N}}$TA was first made to the Second NASTRAN User's Colloquium[1] in September 1972 when this computer program was delivered and its IBM version was installed at GSFC. Effort has since been spent in the verification of the delivered program, debugging and maintaining, application, and new developments. The $\overline{\text{N}}$TA was integrated into the NASTRAN system in its Level 15.5 version which was made available for general use through COSMIC in June 1973. This version contains corrections for coding errors which were detected in post-delivery verification runs. All findings, including possible error fixes, were reported to the NSMO. It is to be noted that the GSFC NASTRAN Level 15.5 IBM-360 version has been updated continually via an in-house effort. Many error corrections and modifications to accommodate new capabilities for both R&D work and flight program support were promptly made to satisfy our immediate needs. These evolutionary changes are identifiable as shown by the last digit following the version labeling 15.5. The current operational version at GSFC is Level 15.5.3.

Our own experience with the $\overline{\text{N}}$TA started since the delivery of the IBM-360 $\overline{\text{N}}$TA to GSFC in June 1972. Test problems were designed to verify program capabilities and to unearth programming defects. At the end of that year, the Colorado Experiment of the OSO-I (OSO-8) was selected as the first flight experiment to test the developed analytical tools.[34,35]

The $\overline{\text{N}}$TA has since been employed to support many scientific instrument packages containing optical systems for various flight programs at GSFC, such as IUE (International Ultraviolet Explorer Satellite),[36] SMM (Solar Maximum Mission), etc. Figures 1.2–1.5 show some thermal models of the telescopes in the OSO-I Colorado Experiment and the IUE.

Though maintaining the NASTRAN system for general users is the responsibility of the NSMO, we at GSFC have assisted $\overline{\text{N}}$TA users whenever they contacted us for advice. $\overline{\text{N}}$TA users have included NASA field installations, other Government agencies, private industry, and universities. Inquiries were primarily related to the use of this program, modeling techniques, thermal analysis in tandem with structural analysis, programming errors, accuracy and efficiency considerations, etc. Applications of the $\overline{\text{N}}$TA by other users known to us have been working in the areas of nuclear reactors, weaponry, computer hardware, railroad cars, oil refineries, and automobiles.

In March 1975, the NSMO established a new service contract with Universal Analytics, Inc. for regular maintenance of the $\overline{\text{N}}$TA. GSFC was requested officially to be a technical consultant on matters concerning the $\overline{\text{N}}$TA.

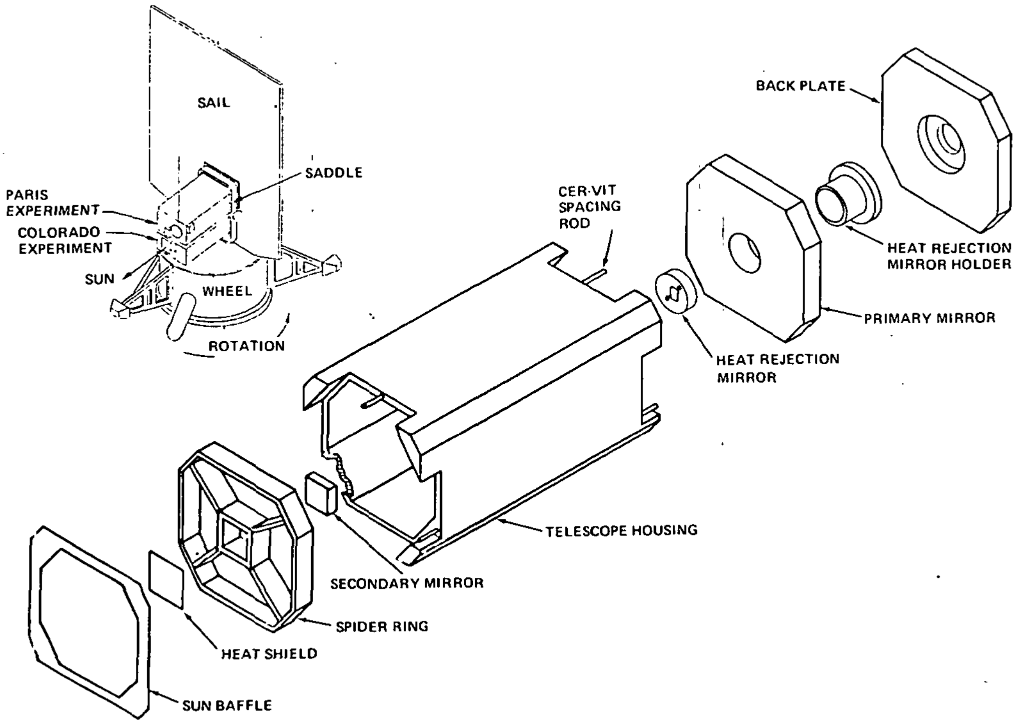


Figure 1.2. The telescope in the OSO-I Colorado experiment.

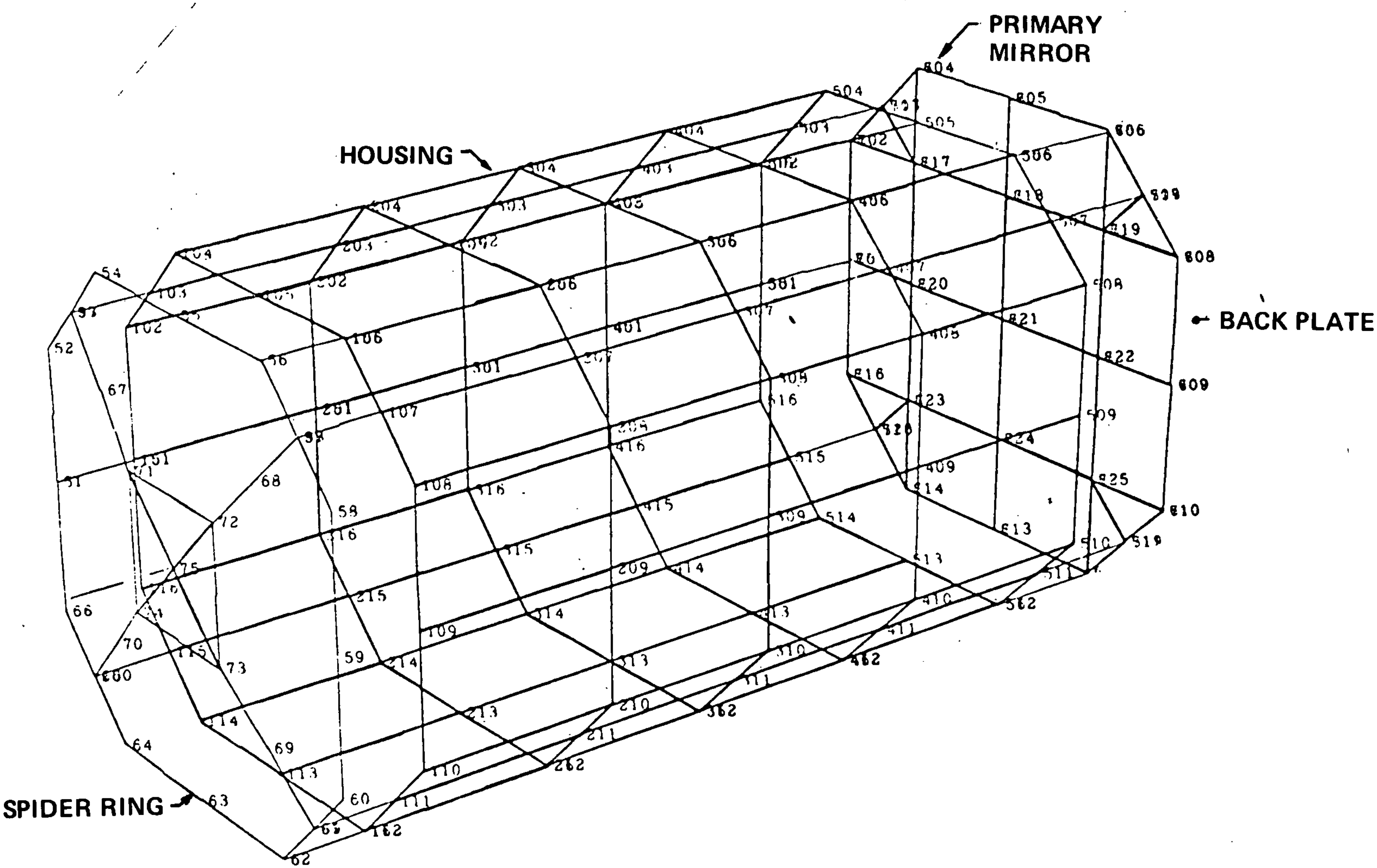


Figure 1.3. Conduction elements in the finite element thermal model of the telescope in the OSO-I Colorado experiment.

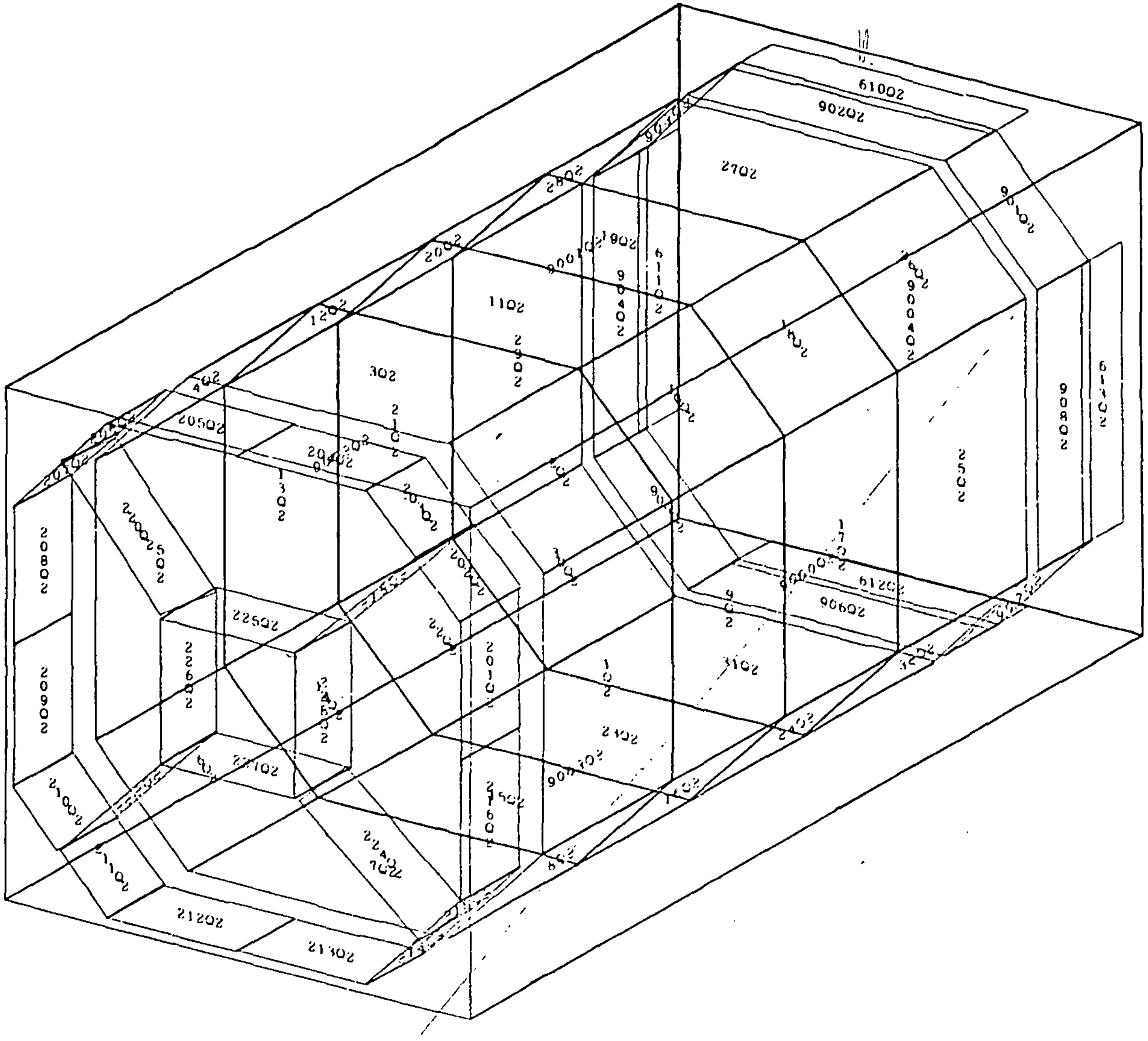
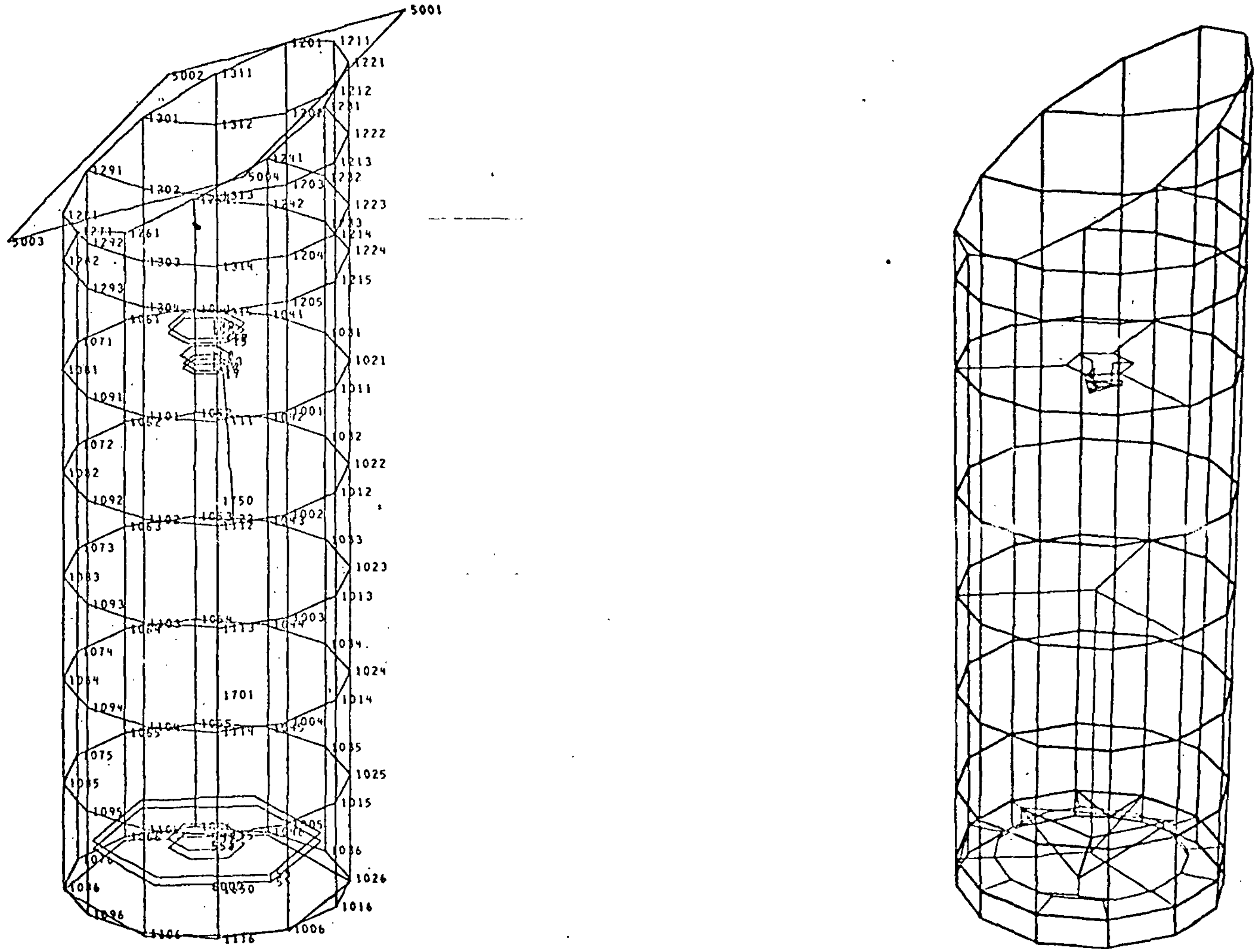


Figure 1.4. Boundary surface elements for radiative exchanges between the telescope and the casing walls of the OSO-I Colorado experiment.



(a) THE BOUNDARY SURFACE ELEMENTS

(b) THE IUE TELESCOPE CONDUCTION ELEMENTS

Figure 1.5. The IUE telescope thermal model.

1.2 Distinctions Between the Finite Difference Method and the Finite Element Method

The widespread use of high speed digital computers has made numerical methods extremely valuable for solving practical engineering problems which are generally not amenable to analytical methods. In contrast to the finite difference method which approximates derivatives in a differential equation, the finite element method applies an approximation to the terms of a variational formulation. The distinctions between these two numerical methods from a user's, rather than a theoretical, view point are to be differentiated in this section.

Figure 1.6 shows the difference between a 2-D conducting medium being discretized by the finite difference method and the finite element method. Although the continuous field and the continuous independent variables are replaced by a discretized system in both numerical methods, each isothermal area is assumed and represented by a lumped node at its center in the case of the finite difference lumped-nodal method, and temperature variables are represented at the vertices of each element in the case of the finite element method.

In dealing with irregular shaped regions, the nodal points of the finite difference method near the boundary have to use separate equations, (e.g., reference 37), other than those representing the interior points, but the finite element method using triangular and quadrilateral elements provide a much better approximation of the same region than is provided by the other method. The finite elements used at the boundary are no different than any interior ones. Consequently, no additional programming effort is required.

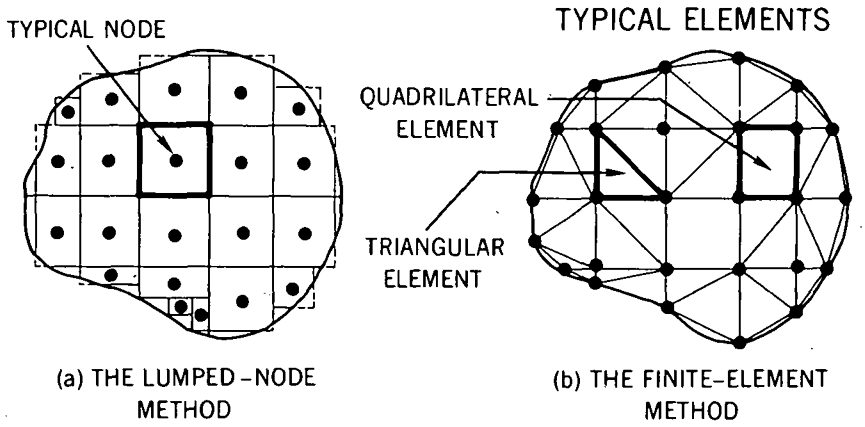


Figure 1.6. Distinction between the finite difference lumped-nodal method and the finite element method.

Table 1.2 compares the saving of modeling effort with the two different approaches when a thermal analysis including radiative exchanges, and a structural analysis, are performed in tandem. Following a conventional approach, three independent models have to be prepared: A model defines discretized isothermal surfaces for generating view factors, another model gives discrete temperature nodes representing the isothermal surfaces for the lumped-nodal network thermal analysis, and a third finite element structural model is used by NASTRAN for a structural analysis. In the unified finite element approach, however, the view factors are generated by a specially developed VIEW program[38,39] which uses the same boundary surface elements as required in the $\overline{N}$TA model for thermal analysis. The structural elements in the NASTRAN structural model are virtually identical to those heat conduction elements used in the $\overline{N}$TA, with the exception of loadings which have to be adjusted or added as required by thermal and structural problems. Only one model is required to be prepared, and the cost-effectiveness is evident.

Table 1.2

Number of model required for thermo-structural analysis including radiative exchanges.

Approach	Conventional Method	Finite-Element Method
View Factor Generation	(Surfaces)	(Finite Elements)
Thermal Analysis	(Nodal-Network)	
Structural Analysis (NASTRAN)	(Finite Elements)	
Total	3 Different Models	1 Unified Model

1.3 The NASTRAN Thermal Analyzer Capabilities

The capabilities of the current operational version of the $\overline{\text{N}}$TA Level 15.5.3 are summarized as follows:

1. A general purpose heat transfer computer program based on the finite element method
2. Conduction together with convective and radiative boundaries
3. Linear and nonlinear cases in transient and steady-state problems
4. Isotropic and anisotropic, temperature-dependent properties (thermal conductivity, and convective film coefficient being available to steady-state cases only)
5. Time-dependent thermal loadings
6. Compatibility with the finite element based structural and optical analysis capabilities (STOP-program)
7. Graphics capabilities including conduction elements, boundary surface elements and time-history temperature and rate of change of temperature curves at a grid point
8. Restart, punch-card or tape output, etc.

New additions that have been developed or are being implemented to enhance the capabilities or convenience of the $\overline{\text{N}}$TA GSFC version are summarized as follows:

1. The new capabilities already developed:
   a. A highly stable explicit integration algorithm suitable for the finite-element transient application[40]
   b. The temperature variance analysis[41]
   c. The plotting of the boundary elements of the HBDY type[41]
   d. A modification to the radiative matrix to accommodate the case of radiative exchanges with mixed diffuse-specular surface characteristics[42]
   e. A modification that condenses the processing procedure for large radiation matrices by factors of as much as 40.
2. The new capabilities and convenience items currently under development:
   a. The condensation of a radiatively nonlinear finite-element thermal model
   b. The entry of multiple boundary condition sets in one submission for execution (i.e., subcases)
   c. The ability to input temperature-dependent emissivities and absorptivities
   d. The one-dimensional thermo-fluid elements.

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2. FINITE ELEMENT THEORY IN HEAT TRANSFER APPLICATIONS

The finite element method is a numerical process by which a continuum with infinite unknown variables can be approximated by an assemblage of a discrete number of elements (subregions) that reduce the problem to a finite number of unknowns. Each element interconnects with others at its end nodes (in 1-D case) or vertices (in 2-D or 3-D case) depending on the configuration of elements. The desire to obtain temperature solutions using the finite element method was stimulated by the widespread acceptance of this method in structural analysis. As the method was originated within structural mechanics, impetus to extend to the thermal field has been due chiefly to the close relationship between these two distinct disciplines when thermal stress or thermal deflection problems are of concern. The intrinsic nature of the finite element method makes it suitable for computer automation; and the ease of varying material properties, model configuration, and the refinement in element representation and boundary conditions are primary advantages possessed by this method. The unified approach of thermal and structural analyses in tandem ensures an automatic and total compatibility of grid point locations, simplifying the generation of temperature data required for subsequent thermo-structural analysis.

This chapter is intended to provide an underlying theoretical basis for understanding the finite element method in heat transfer applications, to show the mathematical analogy between thermal and structural systems and the constituent matrices of basic elements contained in the NASTRAN Thermal Analyzer, and to summarize the distinct approaches and features of the three solution algorithms that have been implemented in the NTA to solve different types of thermal problems.

In contrast to the finite difference method which approximates derivatives in a differential equation directly, the finite element procedure applies an approximation to the terms of a variational formulation. In applying this method of the Rayleigh-Ritz procedure, a variational principle valid over the entire region is postulated, and the desired solution is the one minimizing the functional which is defined by a suitable integration of the unknown quantities over the entire domain. The finite element method deals directly with an approximate minimization of the functional.

In the first section 2.1, the transition from a variational statement to an equivalent governing differential equation is established and the condition necessary for an appropriate integral to be stationary is also given. The application of the calculus of variations within the context of finite element theory, which will be demonstrated on a one-dimensional conducting rod subject to linear boundary conditions in the steady-state case, is presented in section 2.2. The explicit form of the algebraic equations is derived first to show the procedure, then the concise matrix notation simplifying the representation of arrays of algebraic equations are included for a comparison. An extension to the two-dimensional case treating a triangular element with the most general boundary conditions in the transient-state are considered in the following section, 2.3 Since the radiative boundary condition is a phenomenon important to aerospace applications and radiative exchanges occur between 2-D surfaces, it is, therefore, appropriate to discuss the treatments of the radiative effects in this section.

2.1 Variational Principle

The basic problem in the calculus of variations[9] is to determine a function y(x) that minimizes the integral (also called a functional)

$$I(x) = \int_{x_1}^{x_2} F(x, y, y')\, dx \tag{1-1}$$

where $y' = dy/dx$ and F is a function of both y(x) and y′(x) in addition to the independent variable x. In this problem, the values of y(x) at $x_1 = 0$ and at $x_2 = L$ are specified. Therefore, we have

$$y(0) = y_0 \qquad \text{and} \qquad y(L) = y_L \tag{1-2}$$

To find y(x) we shall consider all admissible functions that will satisfy equation (1-2). From all these possible functions, we select the desired one that gives I its minimum value, or equivalently, that makes the functional stationary. These possible functions may be represented by $\tilde{y}(x,\epsilon)$, where

$$\tilde{y}(x,\epsilon) = y(x) + \epsilon\eta(x) \tag{1-3}$$

The function y(x) is the desired function that will minimize I. $\eta(x)$ is a completely arbitrary differentiable function of x with the property that

$$\eta(0) = 0 \qquad \text{and} \qquad \eta(L) = 0 \tag{1-4}$$

Also ϵ is an arbitrary small quantity and $\epsilon\eta(x)$ is called the variation of y(x) and is conventionally denoted by δy, i.e., $\delta y \equiv \epsilon\eta(x)$. The functions $\tilde{y}$ and y are shown in figure 2.1. To ensure that all $\tilde{y}$ will have the values satisfied at both ends, we specify

$$\tilde{y}(0,\epsilon) = y(0) \qquad \text{and} \qquad \tilde{y}(L,\epsilon) = y(L) \tag{1-5}$$

so that the satisfaction of the boundary conditions of equation (1-2) is assured.

Consider the integral that is obtained by substituting y(x) and y′(x) in equation (1-1) with $\tilde{y}(x,\epsilon)$ and $\tilde{y}'(x,\epsilon)$, i.e.,

$$I(\epsilon) = \int_{x_1}^{x_2} F(x, \tilde{y}(x,\epsilon), \tilde{y}(x,\epsilon))\, dx \tag{1-6}$$

This integral is a function of ϵ, since ϵ will remain as a parameter after the integration over x is performed. It is also seen that when $\epsilon=0$ the integral in equation (1-6) reduces to that of

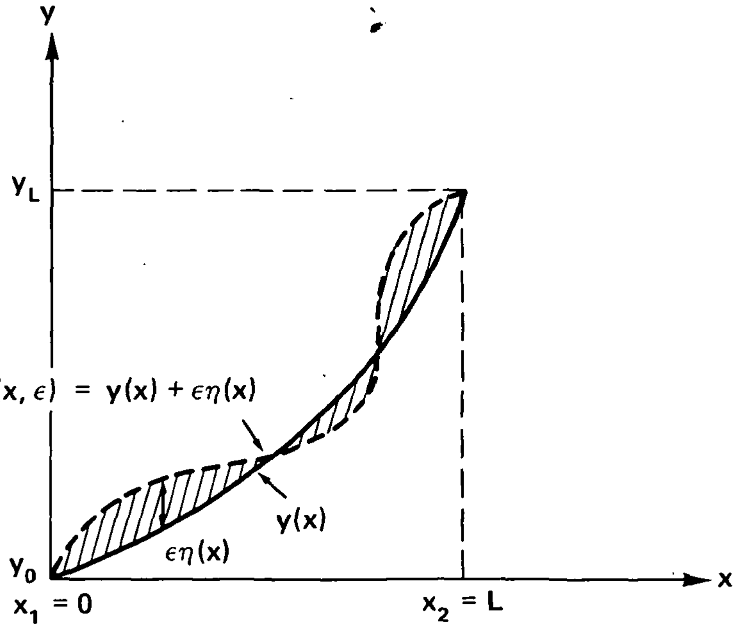


Figure 2.1. True solution y(x) and trial function $\tilde{y}(x, \epsilon)$.

equation (1-1), because $\tilde{y}(x, \epsilon) = y(x)$ when $\epsilon = 0$ as specified by the first given condition in equation (1-5). This means that we want $I(\epsilon)$ to have a minimum when $\epsilon = 0$. Or, it can be restated that for a given $\eta(x)$, I is a function of ϵ, and we require $I(\epsilon)$ to attain a stationary value at $\epsilon = 0$, i.e. $(dI/d\epsilon)_{\epsilon = 0} = 0$.

Expanding $F(x, \tilde{y}(x, \epsilon), \tilde{y}'(x, \epsilon)) = F(x, y(x) + \epsilon\eta(x), y'(x) + \epsilon\eta'(x))$ as a Taylor series about x, y and y'

$$I = \int_{x_1}^{x_2} \left[F\left(x, y(x), y'(x)\right) + \epsilon\left(\eta(x)\frac{\partial F}{\partial y} + \eta'(x)\frac{\partial F}{\partial y'}\right) + \frac{\epsilon^2}{2!}\left(\eta(x)\frac{\partial}{\partial y} + \eta'(x)\frac{\partial}{\partial y'}\right)^2 F + \ldots \ldots\right] dx \quad (1\text{-}7)$$

Differentiating with respect to ϵ,

$$\frac{dI}{d\epsilon} = \int_{x_1}^{x_2} \frac{d}{d\epsilon}\left[F(x, y, y') + \epsilon\left(\eta(x)\frac{\partial F}{\partial y} + \eta'(x)\frac{\partial F}{\partial y'}\right) + 0(\epsilon^2)\right] dx$$

$$= \int_{x_1}^{x_2} \left[\left(\eta(x)\frac{\partial F}{\partial y} + \eta'(x)\frac{\partial F}{\partial y'}\right) + 0(\epsilon)\right] dx$$

where $0(\epsilon)$ implies that the next high-order term is of order ϵ.

The second term may be integrated by parts to give

$$\frac{dI(\epsilon)}{d\epsilon} = \int_{x_1}^{x_2} \eta(x)\,\frac{\partial F}{\partial y}\,dx + \frac{\partial F}{\partial y'}\,\eta(x)\Bigg|_{x_1=0}^{x_2=L} - \int_{x_1}^{x_2} \eta(x)\,\frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right)dx + \int_{x_1}^{x_2} 0(\epsilon)dx \qquad (1\text{-}8)$$

Equation (1–4) indicates that the integrated term in the above expression vanishes at both upper and lower limits. Thus, the first two integrals in the preceding expression can be recombined to give

$$\frac{dI(\epsilon)}{d\epsilon} = \int_{x_1}^{x_2} \eta(x)\left[\frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right)\right]dx + \int_{x_1}^{x_2} 0(\epsilon)dx$$

This integral is required to be zero when $\epsilon = 0$ so that I(0) will be an extremum. From equation (1–3) it may be seen that $\tilde{y}$ has become y because ϵ has been set equal to zero. Thus

$$\left(\frac{dI}{d\epsilon}\right)_{\epsilon=0} = \int_{x_1}^{x_2} \eta(x)\left[\frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right)\right]dx + \int_{x_1}^{x_2} 0(\epsilon)dx = 0 \qquad (1\text{-}9)$$

When $\epsilon = 0$ the high–order terms in ϵ as represented by the second integral in equation (1–9) vanish. Since $\eta(x)$ is arbitrary, the term in the bracket must be zero to ensure that this remainder integral will be zero. Therefore, it is concluded that for I to be a minimum,

$$\frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) = 0 \qquad (1\text{-}10)$$

This differential equation is called the Euler–Lagrange equation. Its boundary conditions in this particular problem have been specified in equation (1–2). Since $\bar{F}(x, y, y')$ is known explicitly for a given problem, the solution y(x) to this differential equation makes the integral I of equation (1–1) stationary. Or it can be stated differently that y(x) satisfying equation (1–10) will be the function that minimizes the original integral or the functional. Since physical problems can always be described by differential equations, the variational formulation of these problems may be obtained by following the steps to reach equation (1–10) in the reverse order. Therefore, starting by treating the differential equation as the Euler–Lagrange equation associated with prescribed boundary conditions, the proper variational formulation can be systematically achieved.

Since our interest is mainly in application of the calculus of variations within the context of finite element theory, we shall employ an illustrative example to show the equivalence of the variational and boundary value problems and to emphasize the physical significance of the variational integral.

2.2 A One-Dimensional Steady-State Conducting Rod

2.2.1 Finite Element Formulation

As an illustration of applying the variational principle to the finite element formulation of a heat transfer problem, let us consider a one-dimensional conducting rod of a constant cross-section A which has its peripheral boundary in contact with an ambient fluid maintained at a constant temperature T_f. A constant convective film coefficient h is assumed. An internal heat source dissipates energy at a constant rate of q_v. A fixed temperature T_b is prescribed at one end of the rod while the other end is assumed to be perfectly insulated. Figure 2.2 depicts this physical problem.

To start, we write the governing differential equation of the problem together with associated boundary conditions as follows

$$kA \frac{d^2T}{dx^2} + q_v A - hp(T - T_f) = 0 \tag{2-1}$$

and

$$\begin{cases} T(x=0) = T_b \\ \left.\dfrac{dT}{dx}\right|_{x=L} = 0 \end{cases} \tag{2-2}$$

where

k is a constant thermal conductivity

L is the length of the conducting rod

p is the perimeter of the rod

T is temperature

x is spatial variable

The other symbols have been defined in the problem description.

In order to deduce the variational statement from the differential equation, we need to determine the equivalent function F of the Euler–Lagrange equation for equation (2–1). When y is replaced by T and y′ by T′ = dT/dx, it is seen by a direct comparison of equation (1–10) with equation (2–1) that

$$\begin{cases} \dfrac{\partial F}{\partial T} = hp(T - T_f) - q_v A \\ \dfrac{\partial F}{\partial T'} = kAT' \end{cases} \tag{2-3}$$

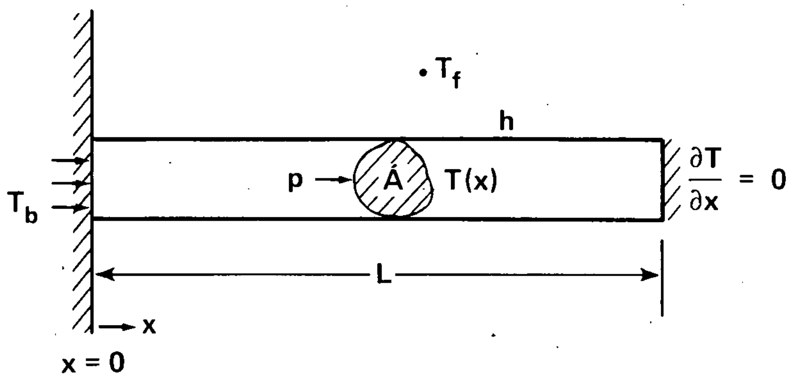


Figure 2.2. Physical representation of a one-dimensional rod.

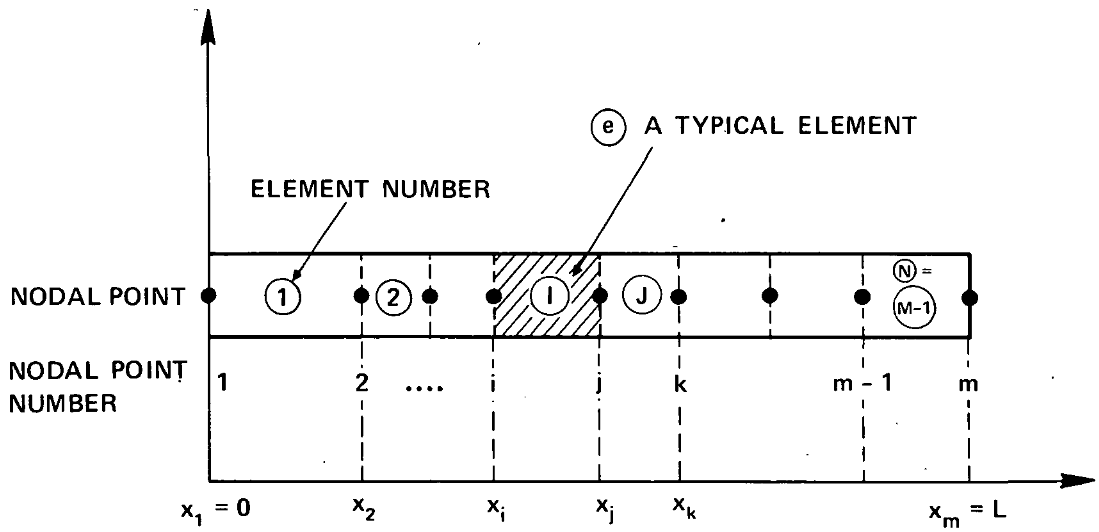


Figure 2.3. Finite element representation of a one-dimensional rod.

Since T and T' are to be treated as independent variables of the function F, each of the above expressions may be partially integrated to give

$$F = hp\left(\frac{1}{2} T - T_f\right)T - q_v AT + f(T')$$

and

$$F = \frac{1}{2} kAT'^2 + g(T)$$

where $f(T')$ and $g(T)$ are two arbitrary functions with respective arguments of T' and T in each of the above two expressions. For F to be unique, these two functions can be deduced by comparison of these two expressions for F, thus

$$f(T') = \frac{1}{2} kAT'^2$$

and

$$g(T) = \frac{1}{2} hp(T - 2T_f)T - q_v AT$$

consequently

$$F = \frac{1}{2} [hp(T - 2T_f)T + kAT'^2] - q_v AT$$

An equivalence to the solution of equation (2–1) may be found by determining the function T(x) which satisfies the same boundary conditions, equation (2–2), and also minimizes the integral

$$I = \frac{1}{2}\int_{x_1 = 0}^{x_2 = L} \left[kA\left(\frac{dT}{dx}\right)^2 + hp(T - 2T_f)T - 2q_v AT\right] dx \qquad (2\text{-}4)$$

It is now possible to use the finite element formulation to obtain an approximate solution for the temperature profile T(x) in the rod.

To start the finite element method, the rod is discretized by specifying nodal point locations along the axis of the rod between the interval x = 0 and x = L as shown in figure 2.3. An element is defined as the subregion between two adjacent nodal points. These nodal points are

numbered from 1 to m, and these elements are numbered from 1 to N. A typical element e represents the subregion bounded by the two end nodes i and j.

After the rod has been discretized by the finite element representation, the evaluation of the integral pertaining to the entire rod, equation (2-4), is equivalent to evaluating subintegrals over each of the N elements, i.e.

$$I = \sum_{e=1}^{N} I^e \tag{2-5}$$

where the subintegral I^e over a typical finite element e is given by

$$I^e = \frac{1}{2}\int_{x_i}^{x_j} \left[k^e A^e \left(\frac{dT^e}{dx}\right)^2 + h^e p^e (T^e - 2T_f)T^e - 2q_v^e A^e T^e\right] dx \tag{2-6}$$

The fact that all parameters k, A, h, p, and q_v, together with the variable T in equation (2-6) are appended by a superscript e, signifies that their values are allowed to be variant from element to element. To proceed with the integral evaluation, the temperature distribution within the element must be assumed. The simplest form is a temperature profile that varies linearly in each element, although other forms of temperature profiles may also be selected. For the purposes of illustration and simplicity, the linear temperature variation will be used. The temperature T^e within the element e is, therefore, represented by

$$T^e = a_1^e + a_2^e x \tag{2-7}$$

The superscript e attached to the two constants a_1 and a_2 also indicates that, in general, these constants can be different from element to element. These constants can be determined by solving equation (2-7) and are expressed in terms of the nodal temperatures T_i and T_j at x_i and x_j, respectively. Thus

$$T_i = a_1^e + a_2^e x_i, \qquad T_j = a_1^e + a_2^e x_j \tag{2-8}$$

The solution of the simultaneous equations yields

$$a_1^e = \frac{x_j T_i - x_i T_j}{x_j - x_i}, \qquad a_2^e = \frac{T_j - T_i}{x_j - x_i} \tag{2-9}$$

Substituting equation (2–9) into equation (2–7), the expression of the temperature distribution within the element is found to be

$$T^e = \frac{1}{x_j - x_i} [(x_j T_i - x_i T_j) + (T_j - T_i)x] \tag{2-10}$$

The derivative of the temperature within the element is

$$\frac{dT^e}{dx} = \frac{T_j - T_i}{x_j - x_i} \tag{2-11}$$

The temperature distribution, equation (2–10), and its derivative, equation (2–11), are substituted into equation (2–6) to yield

$$I^e = \frac{1}{2}\int_{x_i}^{x_j}\left\{k^e A^e\left(\frac{T_j - T_i}{x_j - x_i}\right)^2 + \frac{h^e p^e}{(x_j - x_i)^2} [(x_j T_i - x_i T_j) + (T_j - T_i)x]^2 \right.$$
$$\left. - \frac{2(h^e p^e T_f + q_v^e A^e)}{x_j - x_i} [(x_j T_i - x_i T_j) + (T_j - T_i)x]\right\} dx \tag{2-12}$$

This integral I^e will be a function of T_i and T_j after the integration over x is carried out. All other parameters are known quantities.

The minimization of I^e requires the derivatives of I^e with respect to both T_i and T_j. Since the mathematical operations to be performed on equation (2–12) involve integrations and differentiations, the order in performing such operations is immaterial. The differentiation with respect to T_i or T_j will be performed first and then the integration over x between the interval of x_j and x_i. The resulting expression is obtained as

$$\frac{\partial I^e}{\partial T_i} = \frac{k^e A^e}{x_j - x_i}(T_i - T_j) + \frac{h^e p^e}{6}(x_j - x_i)(2T_i + T_j)$$
$$- \frac{1}{2}(h^e p^e T_f + q_v^e A^e)(x_j - x_i) \tag{2-13}$$

Similarly,

$$\frac{\partial I^e}{\partial T_j} = \frac{k^e A^e}{x_j - x_i}(T_j - T_i) + \frac{h^e p^e}{6}(x_j - x_i)(2T_j + T_i)$$
$$- \frac{1}{2}(h^e p^e T_f + q_v^e A^e)(x_j - x_i) \tag{2-14}$$

Next, a minimum for I over the entire region must be found. Since the subintegral I^e has been shown to be a function of its nodal temperatures T_i and T_j, this indicates that the integral for the entire region, as represented by equation (2–5), will be a function of the complete set of unknown temperatures, i.e.

$$I = I(T_1, T_2, ... T_i, T_j, ... T_m)$$

The minimization of I requires the differentiation of individual subintegrals with respect to each of the nodal temperatures and the setting of each derivative equal to zero. Therefore, m nodal temperatures will result in m equations from this differentiation operation. When equation (2-5) is differentiated with respect to a typical nodal temperature T_j, it gives

$$\frac{\partial I}{\partial T_j} = \frac{\partial I^1}{\partial T_j} + \frac{\partial I^2}{\partial T_j} + ... + \frac{\partial I^N}{\partial T_j} = \sum_{e=1}^{N} \frac{\partial I^e}{\partial T_j} \tag{2-15}$$

It is seen from figure 2.3 that only the two elements I and J in the complete set contain T_j, and the rest of the subintegrals are independent of T_j. Thus, equation (2–15) is reduced to

$$\frac{\partial I}{\partial T_j} = \frac{\partial I^I}{\partial T_j} + \frac{\partial I^J}{\partial T_j} \tag{2-16}$$

Making use of the general expressions derived for the typical element e in equations (2–13) and (2–14), we can evaluate the two derivatives on the right–hand side of equation (2–16) to give

$$\frac{\partial I^I}{\partial T_j} = \frac{k^I A^I}{x_j - x_i} (T_j - T_i) + \frac{h^I p^I}{6} (x_j - x_i)(2T_j + T_i)$$
$$- \frac{1}{2} (h^I p^I T_f - q_v^I A^I)(x_j - x_i) \tag{2-17}$$

and

$$\frac{\partial I^J}{\partial T_j} = \frac{k^J A^J}{x_k - x_j} (T_j - T_k) + \frac{h^J p^J}{6} (x_k - x_j)(2T_j + T_k)$$
$$- \frac{1}{2} (h^J p^J T_f + q_v^J A^J)(x_k - x_j) \tag{2-18}$$

The superscripts I and J identify the subregions or elements with which all known parameters k, A, h, p and q_v are associated. They are carried to emphasize the flexibility of the finite element method which permits entering different values for the same parameter in different

elements. In the present problem, however, since constant parameters have been assumed for the entire region, the superscripts will be omitted from the following expressions.

Substituting equations (2–17) and (2–18) into equation (2–16), we have

$$\frac{\partial I}{\partial T_j} = kA\left(\frac{T_j - T_i}{x_j - x_i} + \frac{T_j - T_k}{x_k - x_j}\right) + \frac{hp}{6}\left[(x_j - x_i)(2T_j + T_i) + (x_k - x_j)(2T_j + T_k)\right]$$

$$- \frac{1}{2}(q_v A + hp\,T_f)\left[(x_j - x_i) + (x_k - x_j)\right] \qquad (2\text{-}19)$$

For I to be an extremum, the above expression is set equal to zero, and therefore

$$\frac{kA}{\Delta x}(-T_i + 2T_j - T_k) + \frac{hp\Delta x}{6}(T_i + 4T_j + T_k) - \Delta x(q_v A + kpT_f) = 0 \qquad (2\text{-}20)$$

where the length of each element has been taken to be equal, i.e. $x_j - x_i = x_k - x_j = \Delta x$. The above expression may be rearranged as

$$-T_i + C_1 T_j - T_k = C_2 \qquad (2\text{-}21)$$

where

$$C_1 = \frac{12kA + 4hp\Delta x^2}{6kA - hp\Delta x^2} \qquad (2\text{-}22)$$

and

$$C_2 = \frac{6\Delta x^2(hp\,T_f + q_v A)}{6kA - hp\Delta x^2} \qquad (2\text{-}23)$$

Equation (2–21) is valid for any of the interior nodes. However, the first element contains a temperature node at x = 0, i.e. T_1, whose value was prescribed in equation (2–2). A substitution of this known temperature T_b into equation (2–21) yields

$$C_1 T_2 - T_3 = C_2 + T_b \qquad (2\text{-}24)$$

where a change of subscripts of T's with $T_i = T_1 = T_b$, $T_j = T_2$ and $T_k = T_3$ has been made to reflect the specific nodal designations as shown in figure 2.3. As for the very last temperature node in the last element N, the node T_m is contained in that element alone. Therefore,

equation (2–17) should be used to obtain the minimization condition for that element. The resulting expression is

$$-2T_{m-1} + C_1 T_m = C_2 \tag{2-25}$$

The established system of algebraic equations as represented by equations (2–21), (2–24) and (2–25) must be solved simultaneously for the nodal temperatures. For m = 5 and N = 4, the system of equations is

$$\begin{array}{ccccc} T_1 & T_2 & T_3 & T_4 & T_5 \end{array}$$
$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & C_1 & -1 & 0 & 0 \\ 0 & -1 & C_1 & -1 & 0 \\ 0 & 0 & -1 & C_1 & -1 \\ 0 & 0 & 0 & -2 & C_1 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{Bmatrix} = \begin{Bmatrix} T_b \\ C_2 + T_b \\ C_2 \\ C_2 \\ C_2 \end{Bmatrix} \tag{2-26}$$

Equation (2–26) is, in fact, the first time that an expression has appeared in the matrix form in this manual. The temperature solution can be obtained by a matrix inversion which will not be demonstrated here. Methods of solution for different types of heat transfer problems specifically suited for computer solution will be discussed in section 2.6. The preceding example has shown the definition of nodal points and elements. The derivation of the coefficients associated with the nodal temperatures produced thermal conductances which include the conductive couplings within the solid conducting rod and the convective couplings between the boundary surfaces and the ambient fluid. Other important terms obtained are thermal forces appearing on the right-hand side of equations (2–21), (2–24) and (2–25). All expressions were derived on an element basis, and the resulting system of algebraic equations was assembled from individual elements to represent the entire region. In a computerized operation, the element matrices are coded to produce individual matrices, and the assemblage of element matrices is performed by an automated matrix assembler. The matrix notation not only is well suited to computer computations but substantially simplifies the development of basic elements. The benefits would be even more evident in treating 2–D or 3–D elements. As a transitional comparison, the previous problem will be expressed in matrix notation in the next section.

2.2.2 Finite Element Formulation in Matrix Notation

In order to clarify the relation between matrix representation as well as its operations and the finite element formulation, the same illustrative problem used in the preceding section will be employed. Following the same steps from equation (2–1), equation (2–7) is reached. It is now to be expressed in matrix form as the product of a row matrix and a column matrix. For a typical element e, the temperature variation within that element is

$$T^e = \lfloor f \rfloor \{a\}^e \tag{2-27}$$

where

$$\lfloor f \rfloor = \lfloor 1 \quad x \rfloor \tag{2-28}$$

and

$$\{a\}^e = \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix}^e \tag{2-29}$$

The components of $\{a\}^e$ can be expressed in terms of the temperatures at the end nodes of this element. The equivalence of equation (2–8) in matrix notation is of the form

$$\begin{Bmatrix} T_i \\ T_j \end{Bmatrix}^e = \begin{bmatrix} 1 & x_i \\ 1 & x_j \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix}^e \tag{2-30}$$

Let us define the temperature vector whose components are temperatures at the end nodes of this typical element to be

$$\{T\}^e = \begin{Bmatrix} T_i \\ T_j \end{Bmatrix}^e \tag{2-31}$$

and also define

$$[N] = \begin{bmatrix} 1 & x_i \\ 1 & x_j \end{bmatrix} \tag{2-32}$$

Equation (2-30) may be simplified as

$$\{T\}^e = [N]\{a\}^e \tag{2-33}$$

The constant vector of coefficients $\{a\}^e$ is determined by

$$\{a\}^e = [N]^{-1}\{T\}^e \tag{2-34}$$

where $[N]^{-1}$, the inverse of $[N]$, is

$$[N]^{-1} = \frac{1}{x_j - x_i} \begin{bmatrix} x_j & -x_i \\ -1 & 1 \end{bmatrix} \tag{2-35}$$

which is a function of the locations of the end nodes of the element alone. A substitution into equation (2–27) from equation (2–34) yields the expression equivalent to equation (2–10), i.e.

$$T^e = \lfloor f \rfloor [N]^{-1}\{T\}^e \tag{2-36}$$

It is seen from equations (2–28), (2–35) and (2–31) that only $\lfloor f \rfloor$ is spatially dependent within the typical element. The differentiation of T^e with respect to x is, therefore,

$$\frac{dT^e}{dx} = \left\lfloor \frac{df}{dx} \right\rfloor [N]^{-1} \{T\}^e \tag{2-37}$$

Substituting equations (2–36) and (2–37) into equation (2–6), we have the integral extending over the entire subregion (element) in the following form

$$\begin{aligned}
I^e = \frac{1}{2} \int_{x_j}^{x_j} \Bigg[& k^e A^e \left(\left\lfloor \frac{df}{dx} \right\rfloor [N]^{-1} \{T\}^e \right)^2 + h^e p^e (\lfloor f \rfloor [N]^{-1} \{T\}^e)^2 - 2h^e p^e\, T_f \lfloor f \rfloor [N]^{-1} \{T\}^e \\
& -2q_v^e A^e \lfloor f \rfloor [N]^{-1} \{T\}^e \Bigg] dx \\
= \frac{k^e A^e}{2} \int_{x_i}^{x_j} & \left(\left\lfloor \frac{df}{dx} \right\rfloor [N]^{-1} \{T\}^e \right)^2 dx + \frac{h^e p^e}{2} \int_{x_i}^{x_j} [(\lfloor f \rfloor [N]^{-1} \{T\}^e)^2 - 2T_f \lfloor f \rfloor [N]^{-1} \{T\}^e]\, dx \\
-q_v^e A^e \int_{x_i}^{x_j} & \lfloor f \rfloor [N]^{-1} \{T\}^e\, dx
\end{aligned} \tag{2-38}$$

It is to be noted that the parameters k, A, h, p and q_v have been moved outside of the integral signs. This is permitted so long as the parameters are constants or if an average value for each parameter over the element can be selected. It must be emphasized, however, that these parameters are permitted to have different values for each element. This flexibility is one of the advantages of the finite element method which allows it to easily accommodate situations in which the parameters vary with position.

The stationary value of the functional can be obtained by taking derivatives of I^e with respect to $\{T\}^e$ and setting the results equal to zero. Differentiating equation (2–38) leads to

$$\begin{aligned}
\frac{dI^e}{d\{T\}^e} = {} & k^e A^e \int_{x_i}^{x_j} [N]^{-1^T} \left\lfloor \frac{df}{dx} \right\rfloor^T \left\lfloor \frac{df}{dx} \right\rfloor [N]^{-1} \{T\}^e\, dx \\
& + h^e p^e \int_{x_i}^{x_j} ([N]^{-1^T} \lfloor f \rfloor^T \lfloor f \rfloor [N]^{-1} \{T\}^e - T_f [N]^{-1^T} \lfloor f \rfloor^T)\, dx \\
& - q_v^e A^e \int_{x_i}^{x_j} [N]^{-1^T} \lfloor f \rfloor^T\, dx
\end{aligned} \tag{2-39}$$

in which the property of the matrix product of

$$\left\lfloor \frac{df}{dx} \right\rfloor [N]^{-1}\{T\}^e \quad \text{or} \quad \lfloor f \rfloor [N]^{-1}\{T\}^e$$

being a scalar quantity has been utilized. As such it permits the order of their presence in a string of products with other matrices within a term to be switched as the operations were performed inside the first and second integral signs in the above expression. In addition, the following relationships, which can be shown by expanding the matrices together with differentiations with respect to the component temperatures of $\{T\}^e$, have also been incorporated. They are

$$\frac{d}{d\{T\}^e} \left\lfloor \frac{df}{dx} \right\rfloor [N]^{-1}\{T\}^e = \left(\left\lfloor \frac{df}{dx} \right\rfloor [N]^{-1} \right)^T = [N]^{-1^T} \left\lfloor \frac{df}{dx} \right\rfloor^T \tag{2-40}$$

and

$$\frac{d}{d\{T\}^e} \lfloor f \rfloor [N]^{-1}\{T\}^e = (\lfloor f \rfloor [N]^{-1})^T = [N]^{-1^T} \lfloor f \rfloor^T \tag{2-41}$$

Since the quantities $\{T\}^e$, $[N]^{-1}$ and its transpose $[N]^{-1^T}$ are independent of x, they can be moved outside of the integral signs to yield the form

$$\begin{aligned} \frac{dI^e}{d\{T\}^e} &= k^eA^e[N]^{-1^T} \int_{x_i}^{x_j} \left\lfloor \frac{df}{dx} \right\rfloor^T \left\lfloor \frac{df}{dx} \right\rfloor dx.[N]^{-1}\{T\}^e \\ &+ h^ep^e\left([N]^{-1^T} \int_{x_i}^{x_j} \lfloor f \rfloor^T \lfloor f \rfloor dx\, [N]^{-1}\{T\}^e - T_f[N]^{-1^T} \int_{x_i}^{x_j} \lfloor f \rfloor^T dx \right) \\ &- q_v^eA^e[N]^{-1^T} \int_{x_i}^{x_j} \lfloor f \rfloor^T dx \end{aligned} \tag{2-42}$$

As a consequence, the integrals in equation (2–42) which remain to be evaluated are substantially simplified. Substituting equations (2–28), (2–31) and (2–35) into equation (2–42), carrying out the integrations, and setting the resulting expression equal to zero, the expression

$$\frac{k^eA^e}{x_j - x_i} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} T_i \\ T_j \end{Bmatrix}^e + \frac{h^ep^e(x_j - x_i)}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{Bmatrix} T_i \\ T_j \end{Bmatrix}^e = (h^ep^e\,T_f + q_v^eA^e) \frac{(x_j - x_i)}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} \tag{2-43}$$

is obtained. The coefficient matrices associated with the temperature vector in the above expression are, respectively, the conduction matrix K_c^e and the convection matrix K_h^e pertaining to the typical element. They can be combined as a single thermal conductance matrix of the form

$$
\begin{aligned}
K^e &= \begin{bmatrix} k_{11}^e & k_{12}^e \\ k_{21}^e & k_{22}^e \end{bmatrix} \\
&= K_c^e + K_h^e \\
&= \left[\frac{k^e A^e}{\Delta x^e} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{h^e p^e \Delta x^e}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \right]
\end{aligned}
\tag{2-44}
$$

The components of K^e are easily formed from the last expression in equation (2–44), i.e.

$$
\begin{aligned}
k_{11}^e &= k_{22}^e = \frac{k^e A^e}{\Delta x^e} + \frac{h^e p^e \Delta x^e}{3} \\
k_{12}^e &= k_{21}^e = -\frac{k^e A^e}{\Delta x^e} + \frac{h^e p^e \Delta x^e}{6}
\end{aligned}
\tag{2-45}
$$

where $\Delta x^e = x_j - x_i$. The thermal force $\{Q\}^e$ corresponding to the term on the right-hand side of equation (2–43) is attributed to the collective effects of the ambient temperature T_f coupled convectively to the element's periphery, and the internal heat source. $\{Q\}^e$ is a combination of the last terms in equations (2–13) and (2–14).

Equation (2–43) can then be expressed in a concise form as

$$[K]^e \{T\}^e = \{Q\}^e \tag{2-46}$$

Elements are to be assembled to represent the entire set for a physical system. This can be easily done by a summation of all elements to cover the entire region, i.e.

$$\sum_e [K]^e \{T\}^e = \sum_e \{Q\}^e \tag{2-47}$$

All quantities in the above matrix equation can be referred to individual element (local) coordinates which, in general, may be different from a common coordinate system of the assembled system called the global coordinates. Since it is often convenient to evaluate the thermal conductance matrices of individual elements in the local coordinate system so as to minimize the

computing effort, it is necessary to introduce an element transformation matrix [E] to change the frame of reference from one to the other coordinate system. Thus by relating

$$\{T\}^e = \begin{Bmatrix} T_i \\ T_j \end{Bmatrix}^e = [E]\{T\} \tag{2-48}$$

where

$$[E] = \begin{array}{c} \begin{bmatrix} 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & 1 & 0 & \cdots & 0 \end{bmatrix} \\ \begin{array}{cccccccc} & & & \uparrow & \uparrow & & & \\ & & & \text{ith column} & \text{jth column} & & & \end{array} \end{array} \tag{2-49}$$

Similarly, the thermal force matrices in the local coordinate system are transformed and assembled to be of the following form in the global coordinate system

$$\{Q\} = \sum_e [E]^{-1}\{Q\}^e \tag{2-50}$$

As the transformation is orthogonal, the transpose of [E] is equal to its inverse, therefore

$$\{Q\} = \sum_e [E]^T\{Q\}^e \tag{2-51}$$

A substitution of equations (2–47) and (2–48) into equation (2–51) leads to

$$\{Q\} = \sum_e [E]^T[K]^e[E]\{T\} \tag{2-52}$$

Defining a thermal conductance matrix of the assembled set in the global coordinate system as

$$[K] = \sum_e [E]^T[K]^e[E] \tag{2-53}$$

Equation (2–52) can be expressed as

$$[K]\{T\} = \{Q\} \tag{2-54}$$

All matrices are now expressed with respect to the global coordinate system. The global thermal conductance matrix, $[K]$, is a symmetric matrix because it is the sum of symmetric components of the symmetric element matrix $[K]^e$, and the nodal temperature matrix $\{T\}$ is independent of the summation. To illustrate this, the same conducting rod as previously considered will be used to expand the matrices in equation (2-54), i.e.

$$\begin{array}{c} \begin{matrix} T_1 & \quad T_2 & \quad\quad T_3 & \quad\quad T_4 & \quad\quad T_5 \end{matrix} \\ \begin{bmatrix} k^1_{11} & k^1_{12} & & & \\ k^1_{21} & k^1_{22} + k^2_{11} & k^2_{12} & & \\ & k^2_{21} & k^2_{22} + k^3_{11} & k^3_{12} & \\ & & k^3_{21} & k^3_{22} + k^4_{11} & k^4_{12} \\ & & & k^4_{21} & k^4_{22} \end{bmatrix} \end{array} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{Bmatrix} = \begin{Bmatrix} q^1 \\ q^1 + q^2 \\ q^2 + q^3 \\ q^3 + q^4 \\ q^4 \end{Bmatrix} \tag{2-55}$$

where $q^e = (h^e p^e T_f + q_v^e A^e)\,\Delta x^e/2$, and the superscripts e associated with k's and q's have been replaced by the specific element identifiers 1, 2, 3 and 4. The above expression can be further reduced because equal intervals were taken for all elements, then

$$\begin{bmatrix} k_{11} & k_{12} & & & \\ k_{12} & 2k_{11} & k_{12} & & \\ & k_{12} & 2k_{11} & k_{12} & \\ & & k_{12} & 2k_{11} & k_{12} \\ & & & k_{12} & k_{11} \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{Bmatrix} = q \begin{Bmatrix} 1 \\ 2 \\ 2 \\ 2 \\ 1 \end{Bmatrix} \tag{2-56}$$

in which all superscripts have been dropped to reflect the fact that all elements have identical geometric and physical properties.

In applying the boundary condition at $x = 0$ with $T_1 = T_b$, care must be taken not to destroy the symmetry of the conductance matrix. This can be achieved by a proper rearrangement of the components of these matrices in equation (2–56), and the resulting expression is shown as follows:

$$\begin{bmatrix} 1 & 0 & & & \\ 0 & 2k_{11} & k_{12} & & \\ & k_{12} & 2k_{11} & k_{12} & \\ & & k_{12} & 2k_{11} & k_{12} \\ & & & k_{12} & k_{11} \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{Bmatrix} = q \begin{Bmatrix} 0 \\ 2 \\ 2 \\ 2 \\ 1 \end{Bmatrix} + T_b \begin{Bmatrix} 1 \\ -k_{12} \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{2-57}$$

The preservation of the symmetry of the conductance matrix will result in efficient matrix operations. Equation (2–57) reduces to equation (2–26) if the entire set of equations is divided by $-k_{12}$, and the last equation multiplied by 2.

It should be noted that the other boundary condition which specifies an insulated end requires no special modeling effort with respect to the resulting expression of equation (2–57).

The finite element formulation of a 1–D conducting rod has been presented in matrix notation. It provides a step-by-step comparison with the derivation in the preceding subsection 2.2.1. The advantages resulting from the matrix representation will be appreciated even more when 2–D and 3–D elements are treated. The formulation of the thermal conductance matrix consisting of the conduction matrix and the convection matrix, and the thermal force matrix including the effect of a linear convective coupling with an ambient at a constant temperature, have been presented. Insofar as radiative exchanges are concerned, they are boundary effects occurring at surfaces of the finite elements, and they will logically be considered in the next section which will treat a 2–D triangular element.

2.3 Finite Element Formulation of a Two-Dimensional Triangular Element

In the preceding section, 2.2, the deduction of an equivalent variational statement from the differential equation describing the 1-D conducting rod has been demonstrated. In addition, the resulting expressions obtained by minimizing the functional of an element following the finite element procedure were presented. It is recognized that the derivation of a specific element is, in essence, the obtaining of the constituent matrices of the heat equation expressed in terms of the coordinates and temperatures at the vertices of the element. In the previous example, the matrices consisted of the heat conductance matrix which is associated with the temperature vector, and the matrix of thermal loads. To extend the problem to a transient case with general boundary conditions including nonlinear radiative exchanges, a triangular element employing two spatial variables (2-D) will be used. The derivation of heat conductance, heat capacitance and thermal load matrices will be presented in subsection 2.3.1, and a detailed treatment of radiative effects for diffuse-grey surfaces will be included in subsection 2.3.2.

2.3.1 Conduction with Linear Boundary Conditions

The general formulation by the finite element method is demonstrated once more but on a two-dimensional transient heat equation in the cartesian form

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial y}\right) + q_v \tag{3-1}$$

with the boundary conditions of prescribed temperature, T_p, prescribed heat flux, q_b, convective heat transfer, q_h, and radiative exchanges. Mathematically, they are

$$\begin{aligned}
T &= T_p && \text{on } s_p \\
-k\frac{\partial T}{\partial n} &= q_b && \text{on } s_b \\
-k\frac{\partial T}{\partial n} &= h(T - T_f) = q_h && \text{on } s_h \\
k\frac{\partial T}{\partial n} &= q_s && \text{on } s_s \\
-k\frac{\partial T}{\partial n} &= q_r && \text{on } s_r
\end{aligned} \tag{3-2}$$

where h denotes the convective heat transfer coefficient and T_f the constant temperature of an ambient fluid.

The last two statements distinguish two types of radiative effects: (1) the total directional radiant flux from distant sources q_s, and (2) the net radiative flux, q_r, emitted from the surface as governed by the Stefan-Boltzmann law of radiation. q_s behaves identically to the

prescribed heat flux q_b with its magnitude determined by

$$q_s = -\sum_{j=1}^{J} \alpha(\bar{n} \cdot \bar{v}_j)\, S_j \qquad (j = 1, 2, \ldots . J) \tag{3-3}$$

where $\bar{v}_j$ is a unit vector of the jth radiant source with a power intensity S_j, and $\bar{n}$ is a unit normal out of the typical surface whose absorptivity is α. Deferring the consideration of the radiative effect of the second type, the differential equation (3–1) together with the rest of the boundary conditions in equation (3–2), can be cast in a variational form

$$I(T) = \int_A \tau \left\{ \frac{1}{2} \left[k_x \left(\frac{\partial T}{\partial x} \right)^2 + k_y \left(\frac{\partial T}{\partial y} \right)^2 \right] - q_v T + \rho c \frac{\partial T}{\partial t} T \right\} dA$$

$$+ \int_{S_b} q_b\, T\, ds + \int_{S_h} h \left(\frac{1}{2} T^2 - T_f T \right) ds - \int_{S_s} q_s T ds + \boxed{} \tag{3-4}$$

where a plate of uniform thickness τ has been assumed, and each boundary surface implies two possible types, i.e., the top bounding face ds = dA = dxdy and the bounding surface along the edge, e.g., ds = τ dx. T can be any admissible continuous function, but only the desired one is chosen to minimize the value of I(T) for the region of interest. The particular extremal T, in turn, is the solution of the original differential equation. The block formed by dotted lines signifies the term that would be contributed by the nonlinear radiative effect which will be treated in the next subsection.

Following the finite element procedure, one divides the entire area into a number of elements. Since the functional I(T) can be presented as the sum of the integrals over the elements, one needs only to examine a single element, figure 2.4. Let the temperature variation in the mth element be given as a linear polynomial in x and y

$$T(x,y,t) = a_1(t) + a_2(t) + a_3(t)y \tag{3-5}$$

which in matrix notation is

$$T = \lfloor f \rfloor \{a\} \tag{3-6}$$

where $\lfloor f \rfloor$ is a row vector indicating the spatial dimensions, and $\{a(t)\}$ is a column vector of coefficients. The coefficients of the polynomial distribution can be determined in terms of dependent variable values at the vertices which define the element. If $\{T(t)\}^e$ denotes the temperatures at those vertices, $\{a\}$ can be related to $\{T\}^e$

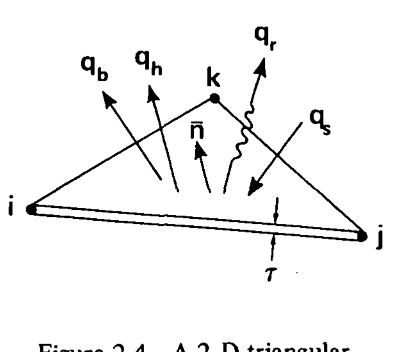


Figure 2.4. A 2-D triangular element.

by the expression of the form

$$\{T\}^e = [N]\,\{a\} \tag{3-7}$$

then

$$\{a\} = [N]^{-1}\,\{T\}^e \tag{3-8}$$

Consequently the temperature profile within the mth element is

$$T = \lfloor f \rfloor\,[N]^{-1}\{T\}^e = \lfloor b \rfloor\{T\}^e \tag{3-9}$$

and

$$\frac{\partial T}{\partial t} = \lfloor b \rfloor \left\{\frac{\partial T}{\partial t}\right\}^e \tag{3-10}$$

where $\lfloor b \rfloor$ is generally termed the shape function or the spatial distribution function.

Since $T(x, y, t)$ is linear, it must be identical for adjoining elements which share the common boundary, hence continuity in T along element interfaces is assured. The temperature gradients can be obtained from equation (3-9), e.g., the x-component is

$$\frac{\partial T}{\partial x} = \left\lfloor \frac{\partial f}{\partial x} \right\rfloor [N]^{-1}\,\{T\}^e \tag{3-11}$$

Substituting equations (3-9) through (3-11) into equation (3-4) and performing the first variation yield the stationary value for I(T), i.e.,

$$\tau \int_{\Gamma} [N]^{-1^T} \left(k_x \left\lfloor \frac{\partial f}{\partial x} \right\rfloor^T \left\lfloor \frac{\partial f}{\partial x} \right\rfloor + k_y \left\lfloor \frac{\partial f}{\partial y} \right\rfloor^T \left\lfloor \frac{\partial f}{\partial y} \right\rfloor \right) [N]^{-1}\,\{T\}^e\,dA$$

$$-\tau \int_{\Gamma} \lfloor b \rfloor^T \left(q_v - \rho c\,\lfloor b \rfloor \left\{\frac{\partial T}{\partial t}\right\}^e \right) dA + \int_{\sigma_b} q_b\,\lfloor b \rfloor^T\,ds$$

$$+ \int_{\sigma_h} h\,\lfloor b \rfloor^T \lfloor b \rfloor \{T\}^e\,ds - \int_{\sigma_h} h\,\lfloor b \rfloor^T\,T_f\,ds - \int_{\sigma_s} q_s\,\lfloor b \rfloor^T\,ds + \boxed{} = 0 \tag{3-12}$$

Equation (3-12), which pertains to a typical element, can be expressed in a concise form as

$$[C]^e\,\{\dot{T}\}^e + [K]^e\,\{T\}^e = \{Q_1\}^e + \boxed{} \tag{3-13}$$

where

$$
\begin{aligned}
[K]^e &= [K_c]^e + [K_h]^e \\
\{Q_l\}^e &= \{Q_v\}^e + \{Q_h\}^e - \{Q_b\}^e + \{Q_s\}^e \\
[C]^e &= \tau \int_{\Gamma} \rho c \lfloor b \rfloor^T \lfloor b \rfloor \, dA \\
[K_c]^e &= \tau \int_{\Gamma} [N]^{-1^T} \left(k_x \left\lfloor \frac{\partial f}{\partial x} \right\rfloor^T \left\lfloor \frac{\partial f}{\partial x} \right\rfloor + k_y \left\lfloor \frac{\partial f}{\partial y} \right\rfloor^T \left\lfloor \frac{\partial f}{\partial y} \right\rfloor \right) [N]^{-1} dA \\
[K_h]^e &= \int_{\sigma_h} h \lfloor b \rfloor^T \lfloor b \rfloor \, ds \\
\{Q_v\}^e &= \tau \int_{\Gamma} q_v \lfloor b \rfloor^T dA \\
\{Q_h\}^e &= \int_{\sigma_h} h \lfloor b \rfloor^T T_f \, ds \\
\{Q_b\}^e &= \int_{\sigma_b} q_b \lfloor b \rfloor^T ds \\
\{Q_s\}^e &= \int_{\sigma_s} q_s \lfloor b \rfloor^T ds
\end{aligned}
\tag{3-14}
$$

It should be noted that the terms that arise from boundary conditions representing the bounding edges exist only for the elements which have one or more of their boundaries coincident with the boundary of the system. Furthermore, if constant values for all these quantities ρ, c, k, h, q_v, q_b and q_s are imposed, they can be brought outside of the individual integral signs in equation (3–14).

Assembling all elements over the entire region to represent a physical body, we obtain the general heat equation expressed in the global coordinates with the limitation of linear thermal loads, $\{Q_l\}$, as

$$[C]\ \{\dot{T}\} + [K]\ \{T\} = \{Q_l\} + \boxed{\quad\quad} \tag{3-15}$$

where

$$[C] = \sum_{e} [E]^T [C]^e [E]$$

$$[K] = \sum_{e} [E]^T [K]^e [E] \tag{3-16}$$

$$\{Q_1\} = \sum_{e} [E]^T \{Q_1\}^e$$

The presence of the element transformation matrix [E] is required to transfer elements from the local coordinate system to the global coordinate system in a matrix assemblage, as explained in section 2.2. In the case of a 2–D triangular element, this transposed element transformation matrix is

$$[E]^T = \begin{array}{cl} \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} & \begin{array}{l} \\ \leftarrow \text{ith row} \\ \\ \leftarrow \text{jth row} \\ \\ \\ \\ \\ \\ \leftarrow \text{kth row} \\ \\ \end{array} \end{array} \tag{3-17}$$

It should be noted that the last expression in equation (3–16) implies that all component thermal loads being applied to the bounding surfaces are of the type where ds = dxdy. If the bounding surfaces are of the type ds = τ dx, [E] will have a different form. In practice, assembling element matrices is always performed by a computer, and therefore only the concept of the transformation of coordinate systems is important rather than the detail of individual transformation matrices.

To complete the deferred term which was denoted by the block formed by dotted lines in equations (3–4), (3–12), (3–13) and (3–15), the nonlinear radiative boundary term will be treated in the next subsection, 2.3.2. While methods of solution for solving problems with a nonlinear radiative boundary are available in a variety of forms[31], only the ones implemented in the NASTRAN Thermal Analyzer will be discussed in section 2.6.

2.3.2 Nonlinear Radiative Boundary Condition

In diffuse-grey thermal radiation analysis, the net radiant energy flux q_r^e emitted from the surface of each element can be determined by an application of the net radiation method

(e.g., reference 43). The usual assumptions pertinent to the diffuse-grey surface are imposed, such as each finite element surface being Lambertian and isothermal, and the incident and hence the reflected energy flux being uniform over each individual surface. To incorporate the resulting expression for a finite element application, an element radiation matrix $[R]^e$ will be formulated first. The net radiant energy will then be properly distributed to the vertices of an element or to the grid points in an assembled system.

The element radiation matrix correlates the radiant flux to the temperature of the element by

$$\lceil A \rfloor \cdot \{q_r\}^e = \{Q_r\}^e = [R]^e \left\{T_e^4\right\} \tag{3-18}$$

where T_e of the element is an average of the temperatures at the vertices of the element measured on an absolute scale.

The net radiant energy Q^e_{rm} leaving the surface of a typical element m is the difference between the radiosity B_m and the incident radiant flux H_m over the element surface area A_m, i.e.,

$$Q^e_{rm} = A_m (B_m - H_m) \tag{3-19}$$

and the radiosity is obtained by a radiant flux balance as

$$B_m = \sigma \epsilon_m T_m^4 + (1 - \epsilon_m) H_m \tag{3-20}$$

in which the grey-body condition $\rho_m = 1 - \epsilon_m$ has been incorporated. ϵ_m and ρ_m denote thermal emissivity and reflectivity, respectively. The incident radiant flux H_m contributed from all other surfaces of radiosities B_n (n = 1, 2, ...N) in a generalized enclosure composed of N discrete radiatively interacting surface areas is

$$H_m = \sum_{n=1}^{N} f_{mn} B_n \tag{3-21}$$

where B and H are temperature-dependent, and f_{mn} is the diffuse view factor between the emitting surface m and the receiving surface n. Multiplying equation (3-21) by the element surface area A_m and making use of the reciprocity rule, i.e., $A_m f_{mn} = A_n f_{nm}$, the following expression is obtained

$$A_m H_m = A_m \sum_{n=1}^{N} f_{mn} B_n = \sum_{n=1}^{N} (A_m f_{mn}) B_n = \sum_{n=1}^{N} (A_n f_{nm}) B_n \tag{3-22}$$

In matrix form equation (3-22) is

$$\lceil A \rfloor \{H\} = [F] \{B\} \tag{3-23}$$

where $[F] = [Af]$ has been substituted. A typical component of this matrix $[F]$ is $F_{mn} = A_n f_{nm}$. The components in $[F]$, therefore, have units of area, and this matrix $[F]$ is symmetric. Equation (3-20) can be rewritten in matrix form as

$$\{B\} = \sigma \lceil \epsilon \rfloor \{T_e^4\} + \lceil I - \alpha \rfloor \{H\} \tag{3-24}$$

where $\lceil \epsilon \rfloor$ and $\lceil \alpha \rfloor$ are diagonal matrices of emissivities and absorptivities, respectively. The simultaneous solution of equations (3-23) and (3-24) yields

$$\{H\} = \sigma \, [\lceil A \rfloor - [F] \; [I - \alpha]]^{-1} \, [F] \, \lceil \epsilon \rfloor \{T_e^4\} \tag{3-25}$$

and

$$\{B\} = \sigma \, [\lceil \epsilon \rfloor + [I - \alpha] \; [\lceil A \rfloor - [F] \, \lceil I - \alpha \rfloor]^{-1} \; [F] \, \lceil \epsilon \rfloor] \{T_e^4\} \tag{3-26}$$

Substituting equations (3-25) and (3-26) into equation (3-19), after the latter equation has been expressed in matrix form, yields

$$\{Q_r\}^e = \sigma \lceil A \rfloor \, [\lceil \epsilon \rfloor - \lceil \alpha \rfloor \, [\lceil A \rfloor - [F] \, \lceil I - \alpha \rfloor]^{-1} \, [F] \, \lceil \epsilon \rfloor] \{T_e^4\} \tag{3-27}$$

A comparison of equation (3-18) with equation (3-27) reveals that the element radiation matrix is

$$[R]^e = \sigma \lceil A \rfloor \, [\lceil \epsilon \rfloor - \lceil \alpha \rfloor \, [\lceil A \rfloor - [F] \, \lceil I - \alpha \rfloor]^{-1} [F] \, \lceil \epsilon \rfloor] \tag{3-28}$$

in which $[F]$ is symmetric, and the above matrix becomes symmetric if $\lceil \epsilon \rfloor = \lceil \alpha \rfloor$.

The net radiative energies $\{Q_r\}^e$, streaming out from the surfaces of individual elements, and the element temperature vector $\{T_e\}$, which consists of the average temperatures of individual radiatively interacting surfaces of elements, must be transformed to the vertices which define the surfaces of participating elements. Thus

$$\{Q_r\} = [G]^T \{Q_r\}^e \tag{3-29}$$

where $\{Q_r\}$ is the net radiative energy matrix whose components are referenced to grid points. The components of the rectangular transformation matrix $[G]$ are simply determined from internal element connectivities. Thus for a 2-D triangular element, it is

$$\lfloor G \rfloor = \frac{1}{3} \lfloor 1, 1, 1 \rfloor \tag{3-30}$$

Similarly, for a 1-D linear element, one half of the total element radiant energy is associated with the grid point at each end. For a 2-D quadrilateral element, overlapping triangles can be used. The transposed matrix is then used to interpolate the element average temperature from the grid point temperatures, $\{T_g\}$, i.e.

$$\{T_e^4\} = [G]\{T_g^4\} \tag{3-31}$$

The fourth power of the temperatures rather than simply the temperatures is chosen for interpolation to ensure that the grid point radiation matrix remains symmetric.

If we relate the vector of radiative energies to grid points, $\{Q_r\}^g$, and the grid point temperatures, $\{T_g\}$, in a relationship similar to that of equation (3-18), then

$$\{Q_r\}^g = [R]^g\{T_g^4\} \tag{3-32}$$

The relationship between the element radiation matrix and the grid point radiation matrix is obtained by combining equations (3-29), (3-18) and (3-31) and then comparing the result with equation (3-32) to give

$$[R]^g = [G]^T [R]^e [G] \tag{3-33}$$

which is also a symmetric matrix if $[R]^e$ is symmetric.

The vector of nonlinear thermal loads $\{Q_r\}^g$ attributed to the radiative boundary condition as given by equation (3-32) can be integrated into equation (3-15) to obtain a general heat equation in matrix form as

$$[C]\{\dot{T}\} + [K]\{T\} = \{Q_l\} + \{Q_r\} \tag{3-34}$$

where

$$\{Q_r\} = \sum_e \{Q_r\}^g \tag{3-35}$$

Equation (3-34) directly represents a transient heat equation. The steady-state equation is obtained when the $\{\dot{T}\} = \{\partial T/\partial t\}$ term in equation (3-34) vanishes. The solution algorithms available in the $\overline{\text{N}}$TA are such that a single solution routine is provided for both linear and nonlinear transient thermal analyses in which $\{Q_l\}$ is allowed to contain components of constant and time-dependent thermal loads, and $\{Q_r\}$ is a nonlinear thermal load vector. Two separate solution algorithms, however, are supplied for steady-state problems of the linear versus nonlinear cases. A direct matrix inversion can solve the linear problem, while an iterative solution algorithm is provided in the $\overline{\text{N}}$TA to solve the problems of the nonlinear type. Details of these solutions will be given in section 2.6.

2.4 An Analogy Between Structural and Thermal Systems

A mathematical analogy can be drawn between the structural and thermal systems, when both the vibration equation and the heat equation are cast in matrix form following the finite element methods. The vibration equation for general transient structural analysis is of the form

$$[M]\{\ddot{u}\} + [B]\{\dot{u}\} + [K]\{u\} = \{P\} + \{N\} \tag{4-1}$$

where

$\{u\}$	is a vector of displacements at grid points
$\{P\}$	is a vector of applied loads that are allowed to be functions of time
$\{N\}$	is a vector of nonlinear loads that may be displacement-dependent
$[K]$	is a symmetric stiffness matrix
$[B]$	is a symmetric matrix of damping coefficients
$[M]$	is a symmetric mass matrix

and the general heat equation, as obtained in equation (3–34), is of the form

$$[B]\{\dot{u}\} + [K]\{u\} = \{P\} + \{N\} \tag{4-2}$$

where

$\{u\}$	is a vector of temperatures at grid points
$\{P\}$	is a vector of applied thermal loads that are allowed to be functions of time
$\{N\}$	is a vector of nonlinear thermal loads that depend on temperature
$[K]$	is a symmetric matrix of heat conduction
$[B]$	is a symmetric matrix of heat capacitance.

An inspection of equations (4–1) and (4–2) reveals that these two equations are identical if the second order differential term, $\{\ddot{u}\}$, is dropped from equation (4–1).

The existence of this analogy has been exploited to extend the capabilities of the NASTRAN program which was originally designed for structural analysis, to allow it to perform full-fledged heat transfer analysis. This development of the NTA was accomplished by making maximum use of existing elements, modules and system capabilities with a minimum of new additions to satisfy the unique requirements posed by thermal problems.

It is to be noticed that some symbols used in equation (4–2) have been deliberately changed from those used in equation (3–34) to conform with those of the structural vibration equation, equation (4–1). Such changes are beneficial to cross-referencing with the three original manuals of NASTRAN. The changed symbols are easily identified merely by comparing equation (4–2) with equation (3–34). Henceforth, the thermo-structural unified symbols are employed in all sections.

The general heat equation implies three classes of problems that require separate solution algorithms. Equation (4–2) directly represents a transient heat equation. $\{P\}$ is permitted to be a constant or a time–dependent thermal load vector, and $\{N\}$ is a nonlinear thermal load vector arising from radiative exchanges or other temperature–dependent variables such as the convective film coefficient. The steady–state equations are obtained readily when $\{\dot{u}\} = \{\partial u/\partial t\} = 0$. However, two separate solution algorithms are required for steady-state problems of the linear and nonlinear cases. The linear case allows only an array of constant conductivity coefficients and a vector of constant thermal loads. The nonlinear case allows a constant or a temperature-dependent conduction matrix and also a vector of nonlinear thermal loads. The different solution algorithms will be further discussed in section 2.6, Methods of Solution.

The analogy between the structural and thermal systems permits a number of elements, modules and formats of input and output from NASTRAN to be used by the $\bar{\mathrm{N}}$TA. For instance, the matrix arrays [K], [B], $\{P\}$ and $\{N\}$, that are input through many of the Bulk Data Cards of the original structural version of NASTRAN, are computed from heat transfer relationships and properties rather than structural ones. However, the matrix assemblers, and applicable solution steps as well as data recoverers are employed in exactly the same manner regardless of whether a structural or thermal problem is being solved. There are some basic terminology differences between these two physical systems. The term "degrees–of–freedom" represents vectorial displacements of three translational and three rotational quantities associated with each grid point in the structural system, in contrast to the term "temperature variable" which represents a single scalar quantity at each grid point in the thermal system. Since a number of terms used in the $\bar{\mathrm{N}}$TA either in the Case Control Deck or in the titles of output results are directly borrowed from NASTRAN, a list equivalencing the corresponding physical meanings of individual symbols and terminologies is given as follows:

Mathematical analogy between structural and thermal systems

Structural System	Symbol	Thermal System
displacement (6 degrees–of–freedom, 3 translational and 3 rotational)	u	temperature (one degree-of-freedom or generalized coordinate)
velocity	$\dot{u}$	rate change of temperature
acceleration	$\ddot{u}$	–
gradient	∇u	temperature gradient
stiffness	K	conductance
damping	B	heat capacitance
mass	M	–

Structural System	Symbol	Thermal System
applied force	P	thermal load
nonlinear force	N	nonlinear thermal load
strain energy function	U	thermal potential function

2.5 Constituent Matrices for Various Elements

Having shown the similarity between structure and thermal systems by the mathematical analogy in the preceding section, the identified element matrices and applicable functional modules can now be used for these equivalent terms in thermal application either directly or with some modifications. Detailed expressions of element matrices for all types of elements available to the $\overline{\text{N}}$TA are still needed in order to determine the numerical values needed to fill all components of individual matrices. If the structural counterparts do not exist, these expressions also serve as a basis for implementing new ones in order to satisfy requirements posed by the thermal system.

Element matrices are grouped according to function rather than type of element and only the essential steps or resulting expressions are shown rather than the detailed derivations.

2.5.1 Heat Conduction Elements

The basic heat conduction elements available in the NASTRAN Thermal Analyzer consist of 1-D rod element, 2-D triangle, and 3-D tetrahedron. The quadrilateral is composed of overlapping triangles, and the wedges and hexahedron are formed from subtetrahedra, figure 2.5.

2.5.1.1 Thermal Conduction Matrices

The derivations of thermal conduction matrices for the 1-D and 2-D heat conduction elements have been shown in sections 2.2 and 2.3, respectively. The thermal conduction matrix of a heat conduction element may also be derived from a thermal potential function in the same way that the stiffness matrix of a structural element is derived from the strain energy function. The thermal potential function is

$$U = -\frac{1}{2}\int_V \tilde{q} \cdot \nabla u \, dV \tag{5-1}$$

where $\tilde{q}$ is the heat flux density, ∇u is the temperature gradient, and the integration is performed over the volume, V, of the element. The components of the heat flux are related to the components of temperature gradient by

$$q_i = -\sum_j k_{ij} \frac{\partial u}{\partial x_j} \tag{5-2}$$

where k_{ij} is a component of the material conductivity matrix and the index j is summed over the dimensions of the space (one, two, or three dimensions). Using equation (5-2), equation (5-1) may be expressed in matrix form as

$$U = \frac{1}{2}\int_V \left\lfloor \frac{\partial u}{\partial x_i} \right\rfloor [k_{ij}] \left\{ \frac{\partial u}{\partial x_j} \right\} dV \tag{5-3}$$

The temperature, u, at an interior point is a linear combination of the temperatures at the vertices of the element $\{u\}$, i.e.,

$$u = \lfloor L_e \rfloor \{u\}^e \tag{5-4}$$

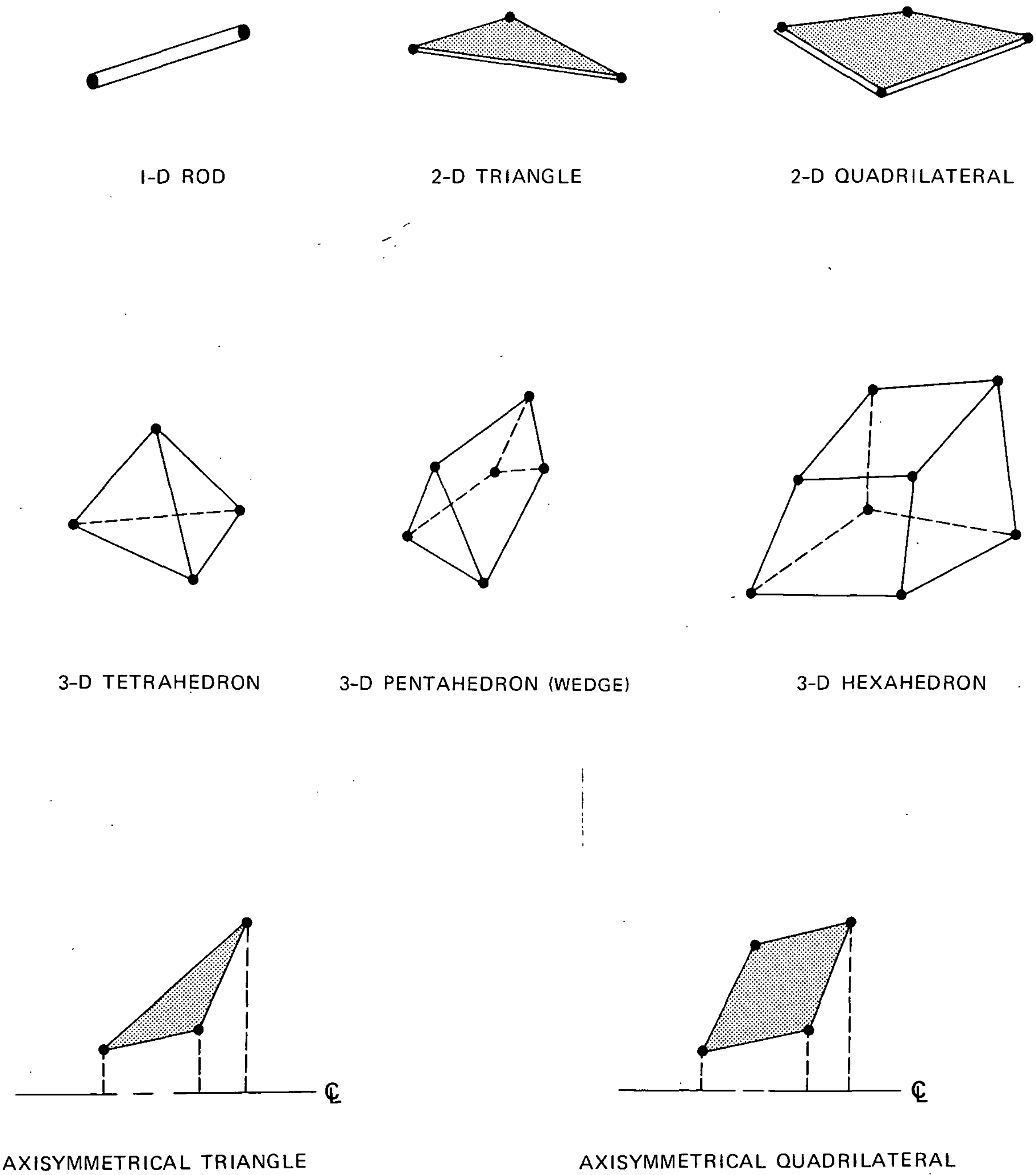


Figure 2.5. Representative heat conduction elements.

where, in general, the components of the row vector $\lfloor L_e \rfloor$ are functions of position. $\lfloor L_e \rfloor$ is the shape function of the temperature distribution and is identical to the matrix $\lfloor b \rfloor$ for a 2–D case as appeared in equation (3–9). The thermal gradient vector is, therefore,

$$\left\{\frac{\partial u}{\partial x_j}\right\} = [L_{e,j}]^T \{u\}^e \tag{5-5}$$

where the derivative matrix $[L_{e,j}]$, for the case of a two-dimensional triangular element, is

$$[L_{e,j}] = \begin{bmatrix} \dfrac{\partial L_1}{\partial x} & \dfrac{\partial L_1}{\partial y} \\ \dfrac{\partial L_2}{\partial x} & \dfrac{\partial L_2}{\partial y} \\ \dfrac{\partial L_3}{\partial x} & \dfrac{\partial L_3}{\partial y} \end{bmatrix} \tag{5-6}$$

In general the number of rows and columns of $[L_{e,j}]$ is the number of vertices of the element, and the dimension of the space, respectively. Therefore, the subscripts (1, 2 and 3) associated with the derivatives of L with respect to the spatial coordinates (x and y) in equation (5–6) identify the vertices of the element, figure 2.6(b). The substitution of equation (5–5) into equation (5–3) produces an expression with the form

$$U = \frac{1}{2} \lfloor u \rfloor^e [K]^e \{u\}^e \tag{5-7}$$

where the element thermal conduction matrix is

$$[K]^e = \int_V [L_{e,i}] [k_{ij}] [L_{e,j}]^T dV \tag{5-8}$$

Equation (5–8) is a general form that is valid for all cases.

For the special case of a constant thermal gradient element with homogeneous properties, $[L_{e,i}]$ and $[k_{ij}]$ in equation (5–8) are constant within the element, so that

$$[K]^e = V_e [L_{e,i}] [k_{ij}] [L_{e,j}]^T \tag{5-9}$$

where V_e is the volume of the element. There is only one general type of constant thermal gradient element for each type of space, i.e., a line segment for a one-dimensional space, a triangle for a two-dimensional space, and a tetrahedron for a three-dimensional space. In the constant thermal gradient case, the vector $\{L_e\}$ is preferably expressed in the "natural

(area) coordinate system"[44]. A single natural coordinate is a linear function spanning the domain of an element with the requirement that it vanishes at all vertices of the element except one whose value is unity. The natural coordinates are obtained by the solution of

$$[H]\ \{L_e\} = \{f\} \tag{5-10}$$

where the specific forms for one, two and three dimensions are

one dimension (line segment)

$$\begin{bmatrix} 1 & 1 \\ x_1 & x_2 \end{bmatrix} \begin{Bmatrix} L_1 \\ L_2 \end{Bmatrix} = \begin{Bmatrix} 1 \\ x \end{Bmatrix} \tag{5-11}$$

two dimensions (triangle)

$$\begin{bmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{bmatrix} \begin{Bmatrix} L_1 \\ L_2 \\ L_3 \end{Bmatrix} = \begin{Bmatrix} 1 \\ x \\ y \end{Bmatrix} \tag{5-12}$$

three dimensions (tetrahedron)

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \\ z_1 & z_2 & z_3 & z_4 \end{bmatrix} \begin{Bmatrix} L_1 \\ L_2 \\ L_3 \\ L_4 \end{Bmatrix} = \begin{Bmatrix} 1 \\ x \\ y \\ z \end{Bmatrix} \tag{5-13}$$

The equivalence of $[H] = [N]^T$ is immediately recognized when equations (3-9) and (5-10) are compared. The determinant of the [H] matrix has a useful property, namely that:

for one dimension, $\det [H] = \ell$, the length of the line segment

for two dimensions, $(1/2) \det [H] = A$, the area of the triangle

for three dimensions, $(1/6) \det [H] = V$, the volume of the tetrahedron

In order to obtain the derivatives of $\{L_e\}$ required in equation (5-9), we observe that, for the two-dimensional case,

$$\{L_e\} = [H]^{-1} \begin{Bmatrix} 1 \\ x \\ y \end{Bmatrix} \tag{5-14}$$

where $[H]^{-1}$ is a matrix of constant coefficients. The derivative matrix may, by comparing

equations (5–6) and (5–14), be expressed formally as

$$[L_{e,j}] = [H]^{-1} \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{5-15}$$

which means that $[L_{e,j}]$ is equal to the last two columns of $[H]^{-1}$. In general, for a space of (n) dimensions, $[L_{e,j}]$ is equal to the last (n) columns of $[H]^{-1}$.

For the case of the tetrahedron, the [H] matrix is inverted numerically, $[L_{e,j}]$ is taken to be the last three columns of $[H]^{-1}$, and $[K]^e$ is evaluated numerically from equation (5–9). All calculations are performed in the local (basic) coordinate system. For one- and two-dimensional elements it is more practical to write explicit formulas for the natural coordinates. In fact, for 1–D elements the thermal conduction matrix, as shown previously in the first term of equation (2–43), is simply

$$[K]^e = \frac{Ak}{\ell} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{5-16}$$

where A is the cross–sectional area, k is the thermal conductivity, and ℓ is the length of the element.

In the case of a 2–D triangular element, the x–axis is taken along the side connecting the two vertices 1 and 2 as shown in figure 2.6(b).

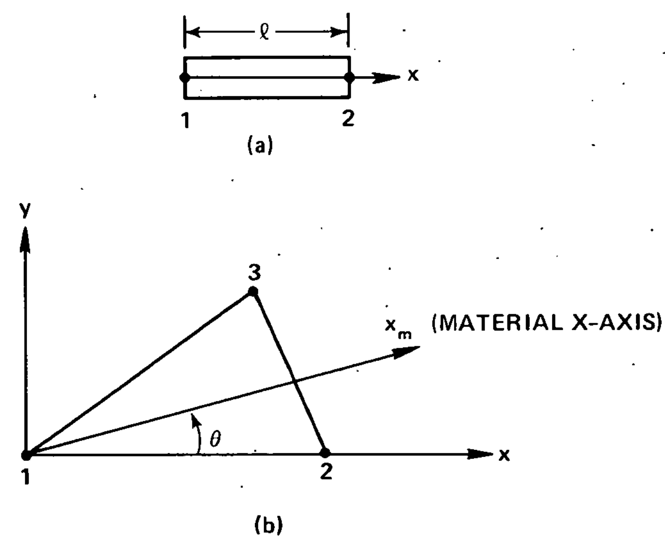


Figure 2.6. Definition of a 1-D rod element and a 2-D triangular element.

The natural coordinates are, by inspection,

$$L_1 = 1 - \frac{x}{x_2} + \left(\frac{x_3}{x_2} - 1\right)\frac{y}{y_3}$$

$$L_2 = \frac{x}{x_2} - \frac{x_3}{x_2}\,\frac{y}{y_3} \tag{5-17}$$

$$L_3 = \frac{y}{y_3}$$

and the derivative matrix is

$$[L_{e,j}] = \left[\begin{array}{c|c} -1/x_2 & (x_3 - x_2)/x_2 y_3 \\ \hline 1/x_2 & -x_3/x_2 y_3 \\ \hline 0 & 1/y_3 \end{array}\right] \tag{5-18}$$

The material conductivity matrix $[k^m]$ is specified in the material coordinate system which makes an angle θ with respect to the element coordinate system as shown in figure 2.6(b). The material conductivity matrix referred to the element coordinate system is

$$[k_{ij}] = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} [k^m] \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \tag{5-19}$$

Equations (5-18) and (5-19) are used in equation (5-9) to obtain the thermal conduction matrix for a triangular plate element. The volume, V_e, is equal to the product of the surface area and the thickness.

For the triangular solid of revolution element (TRIARG) the differential volume to be used in equation (5-8) is $2\pi r\, dr\, dz$, where r and z are cross-sectional coordinates. The temperature is assumed to be constant in the circumferential direction and to vary linearly over the cross-section. Thus, equation (5-8) becomes

$$[K]^e = [L_{e,i}]\,[k_{ij}]\,[L_{e,j}]^T\, 2\pi \int r\,dA$$

$$= \frac{2\pi}{3}(r_1 + r_2 + r_3)\, A_e\, [L_{e,i}]\,[k_{ij}]\,[L_{e,j}]^T \tag{5-20}$$

where A_e is the cross-sectional area. Equation (5-20) is identical to equation (5-9) since the volume of a triangular ring is exactly

$$V_e = \frac{2\pi}{3}(r_1 + r_2 + r_3)\, A_e \tag{5-21}$$

Quadrilateral plate and revolution elements are formed by overlapping triangular elements in exactly the manner described in section 5.8.3.1 of the NASTRAN Theoretical Manual. Hexahedra and wedges are formed from subtetrahedra as described in section 5.12.6 of the same manual. The formation of the quadrilateral plate element by four triangles is shown in figure 2.7.

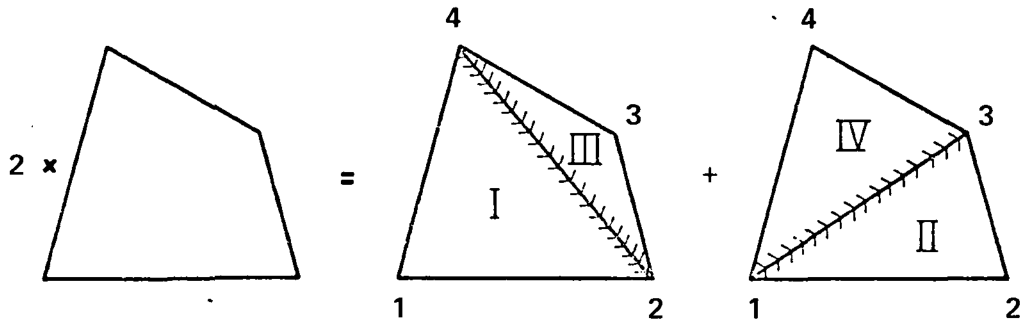


Figure 2.7. Formation of a 2-D quadrilateral element.

2.5.1.2 Heat Capacitance Matrices

In transient thermal analysis, a heat capacitance matrix is generated together with the heat conductivity matrix by each of the basic heat conduction elements. For efficiency and expediency of the computer processing, the heat capacitance matrices [B] used in the NASTRAN Thermal Analyzer are calculated by the lumped method instead of using the consistent matrices. The latter type was discussed in section 2.3.1 for the 2-D triangular element as given by $[C]^e$ in equation (3-14). The resulting expressions for the consistent and lumped heat capacitance matrices for homogeneous elements are given below for comparison. For the 1-D rod element, the resulting consistent heat capacitance matrix is

$$[C]_c = \frac{\rho c A \ell}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \qquad (5\text{-}22)$$

whereas the lumped heat capacitance matrix, which has lumped its thermal capacity at the end points in two equal parts, is

$$[C]_L = \frac{\rho c A \ell}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad (5\text{-}23)$$

For the 2-D triangular element

$$[C]_c = \frac{\rho c A t}{12}\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \qquad (5\text{-}24)$$

and

$$[C]_L = \frac{\rho c A t}{3}\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} \tag{5-25}$$

For the 3-D tetrahedral element

$$[C]_c = \frac{\rho c V}{20}\begin{bmatrix}2 & 1 & 1 & 1\\1 & 2 & 1 & 1\\1 & 1 & 2 & 1\\1 & 1 & 1 & 2\end{bmatrix} \tag{5-26}$$

and

$$[C]_L = \frac{\rho c V}{4}\begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix} \tag{5-27}$$

where ρ is the density of the material, c is the heat capacity per unit volume, A is the cross-sectional area of a rod in the 1-D case and the surface area of a triangle in the 2-D case, ℓ is the length of a rod element, t is the thickness of a triangular element, and V is the volume of a tetrahedron.

In the case of triangular solid of revolution elements, the heat capacitance lumped at the three grid circles is selected so that the total heat capacitance and its center of gravity in the transverse plane are preserved. The equation for the heat capacitance lumped at grid circle (i) is

$$b_i = \frac{\pi c A_e}{6}(r_i + r_1 + r_2 + r_3) \qquad i = 1,2,3 \tag{5-28}$$

The heat capacitance matrices of elements formed by overlapping triangles or tetrahedra are computed by assigning one-half of the capacity to each overlapping set of sub-elements.

Thermal gradients are produced as part of the output, using equation (5-5) and the various expressions derived above for the derivative matrix, $[L_{e,j}]$. The components of the heat flux are also output, using equation (5-2) and the thermal gradient vector.

The temperature gradient and the heat flux are, of course, assumed constant over each subelement. In the case of overlapping elements, a weighted average is computed. The areas of the subtriangles are used as weighting functions in the case of planar elements, and the volumes of the subelements are used as weighting functions in the case of solids.

2.5.2 Boundary Surface Elements

Boundary surface elements are designed and provided to accept external thermal loads including radiative exchanges. If desired, a boundary surface element (HBDY card) may be overlaid

on a surface of a conduction element by connecting it to the same grid points which define the conduction element.

Four types of external thermal loading conditions are considered for both steady-state and transient thermal analyses. These boundary conditions are: (1) a prescribed heat flux, (2) a convective heat transfer due to the temperature difference between the surface and the local ambient condition, (3) a directional radiant flux from a distant source, and (4) the net radiative flux emitting from the surface of the element as governed by the Stefan-Boltzmann law of radiation. In all cases the heat flux is applied to a boundary surface element defined by grid points. There are six distinct types of boundary surface elements:

1. POINT, a flat disc defined by a single grid point
2. LINE, a rectangle defined by two grid points
3. REV, a conical frustrum defined by two grid circles
4. AREA3, a triangle
5. AREA4, a quadrilateral
6. ELCYL, an elliptic cylinder defined by two grid points. Its use is restricted to prescribed directional radiant flux application.

The user is required to supply the area A, for POINT, and a width, w, for LINE. For ELCYL, the principal radii of the cross-section of an ellipse must be given. The surface area is calculated automatically in all other cases.

2.5.2.1 Prescribed Heat Flux

A uniform heat flux, Q, is to be specified by a user, and the vector of the rate of heat flows $\{P\}^e$ is calculated by the program and applied to the grid points connected to an element. The general expression for the jth component of $\{P\}^e$ is

$$P_j^e = A_j^e\, Q_j^e \tag{5-29}$$

where A_j^e is a subarea of the element associated with its jth vertex and Q_j^e is the heat flux at the jth vertex. There are two options for assigning heat fluxes to elements. In the first option via the QBDY1 card, the user specifies a heat flux that is constant over the surface of the element. In the second option via the QBDY2 card, the user specifies separate heat fluxes at the vertices of the element, which are then used directly in equation (5-29). In transient analysis, the time dependence of the flux is specified on a TLOADi card. The subareas A_j^e are calculated in the same manner as heat capacities. Thus, for LINE, A_j^e is one-half of the width multiplied by the distance between the end points, and for AREA3, A_j^e is equal to one-third of the total area. For AREA4, A_j^e is computed from the areas of the overlapping subtriangles connected to the jth grid point. For REV the total area is distributed to the two end points so as to preserve its center of gravity. ELCYL is not available for prescribed heat flux.

2.5.2.2 *Convective Heat Flow*

The rate of convective heat flow into an element's grid points is determined by the general relationship

$$\{P\}^e = [K_h]\{T_f - u_e\} \tag{5-30}$$

where $\{T_f - u_e\}$ is the difference between ambient and surface temperatures at the vertices of the element. The convection matrix $[K_h]$ is calculated as follows for each surface element type. In the following equations, h is the convective film coefficient, which is allowed to be a function of temperature.

POINT:

$$K_h = hA \tag{5-31}$$

LINE:

$$[K_h] = \frac{hw\ell}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \tag{5-32}$$

REV:

$$[K_h] = \frac{\pi h\ell}{6}\left[\begin{array}{c|c} 3r_1 + r_2 & r_1 + r_2 \\ \hline r_1 + r_2 & r_1 + 3r_2 \end{array}\right] \tag{5-33}$$

AREA3:

$$[K_h] = \frac{hA}{12}\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \tag{5-34}$$

where A is the area of the triangle

AREA4:

$$(K_h)_{ij} = \frac{h}{24}\left[(1 + \delta_{ij})(a_1 + a_2 + a_3 + a_4) - (a_i + a_j)\right] \tag{5-35}$$

where

$$\delta_{ij} = \begin{cases} 1 \text{ if } i = j \\ 0 \text{ if } i \neq j \end{cases}$$

a_i = area of the subtriangle which does not touch vertex (i)

The convection matrices are, in each case, derived under the assumption that the temperature difference varies linearly over the surface of the element, except that, in the case of the quadrilateral (AREA4), the temperature difference is assumed to vary linearly over the surface of each overlapping subtriangle.

2.5.2.3 *Directional Radiant Flux From a Distant Source*

The directional radiant flux from a distant source, such as the sun, can be treated as a prescribed heat flux. The magnitude of radiant flux flowing into a boundary surface element depends upon the orientation of the radiation vector relative to the element. The total heat to a single element from a single distant source is given by

$$P = -\alpha A\,(\bar{v}\cdot\bar{n})^{*}Q_0 \tag{5-36}$$

where

P = rate of heat flow into the boundary surface element from the distant radiant source

Q_0 = rate of heat flux of the radiant source

A = surface area of the element

$\bar{v}$ = unit vector of radiant flux (the source is so distant that rays are parallel)

$\bar{n}$ = outward normal to surface

α = absorptivity (if $\alpha < 1$, it is assumed that the reflected radiation is lost from the system)

$(\bar{v}\cdot\bar{n})^{*}$ is replaced by zero in the equation when $\bar{v}\cdot\bar{n}$ is positive, i.e., when the radiation comes from behind the surface.

No provision is made for shading by other surface elements.

In addition to the POINT, LINE and AREA elements, the elliptic cylinder element, ELCYL, can receive prescribed vector radiation, as shown in figure 2.8. An integration of the normal component of flux over the surface is needed to compute the rate of heat flux. The result of the integration is

$$P = 2Q_0\ell\left[(v_y n_y)^2 R_2^2 + (v_z n_z)^2 R_1^2\right]^{1/2} \tag{5-37}$$

where v_y, v_z are components of $\bar{v}$; n_y, n_z are components of $\bar{n}$; and ℓ is the length of the cylinder.

In transient thermal analysis the flux in the incident beam, Q_0, and the components of $\bar{v}$ may be prescribed functions of time. The latter provision is useful in the analysis of rotating spacecraft.

2.5.2.4 *Radiative Exchanges Between Surfaces*

The relationship between the vector of radiative heat flows, $\{Q_r\}^g$, into grid points and the

grid point temperatures, $\{u_g\}$, can be expressed as

$$\{Q_r\}^g = -[R]^g \{(u_g + T_a)^4\} \tag{5-38}$$

In this equation, if $\{T\}$ is a vector, $\{T^4\}$ is defined as the vector whose components are the fourth power of the elements of $\{T\}$. The addition of T_a converts u_g in an engineering temperature scale to an absolute temperature scale. The radiation matrix $[R]^g$ in reference to grid points has been related to the element radiation matrix $[R]^e$ as given in equation (3-33), therefore, no repetition need be made.

The net heat flow into the boundary surface element due to radiation, which is available as output from the NASTRAN Thermal Analyzer, is

$$\{Q\}^e = -[R]^e [G] \{(u_g + T_a)^4\} \tag{5-39}$$

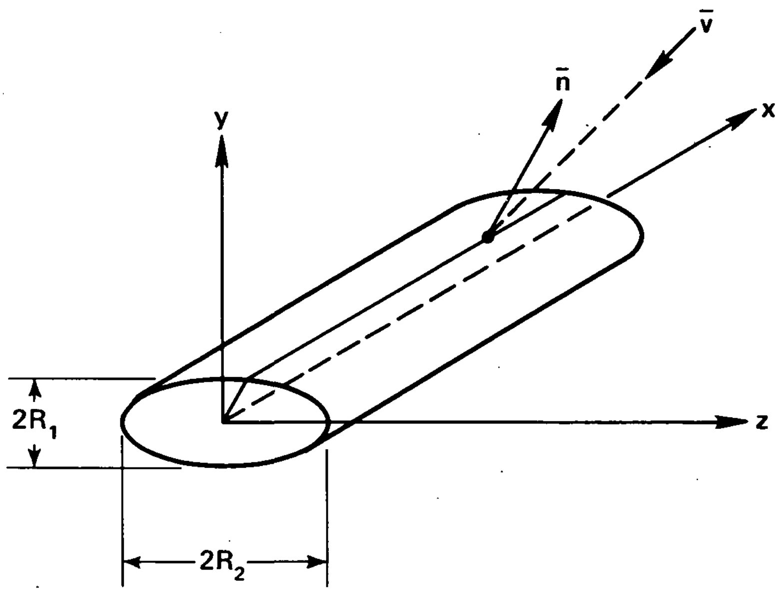


Figure 2.8. An ELCYL type (elliptic cylinder) of the HBDY element.

2.6 Methods of Solution

Heat transfer problems that are solved with the $\overline{\text{N}}$TA are grouped into three types: (1) Linear steady-state thermal analysis, (2) Nonlinear steady-state thermal analysis, and (3) Transient thermal analysis. Both linear and nonlinear options are available in transient thermal analysis.

Flow diagrams for all three types of thermal analysis are shown in figures 2.9, 2.10 and 2.11. Special features of the solutions are described in the subsections that follow.

2.6.1 Linear Steady-State Thermal Analysis

Linear steady-state thermal analysis uses the rigid format (APP HEAT, SØL 1) which is basically the NASTRAN statics rigid format (format No. 1). The principal additions are subroutines for generation of the heat conduction matrix and matrices to facilitate the applications of thermal loads directly to the grid points or indirectly through the boundary surface elements.

Figure 2.9 shows a simplified flow diagram for the linear steady-state thermal analysis which is identical to the structural rigid format 1. Each block in the flow diagram represents a number of subprograms and/or modules. The total number of modules called is approximately thirty. A brief description for each functional block is given below:

The Input File Processor, as the name implies, reorganizes the information on input data cards into Data Blocks consisting of lists of similar quantities.

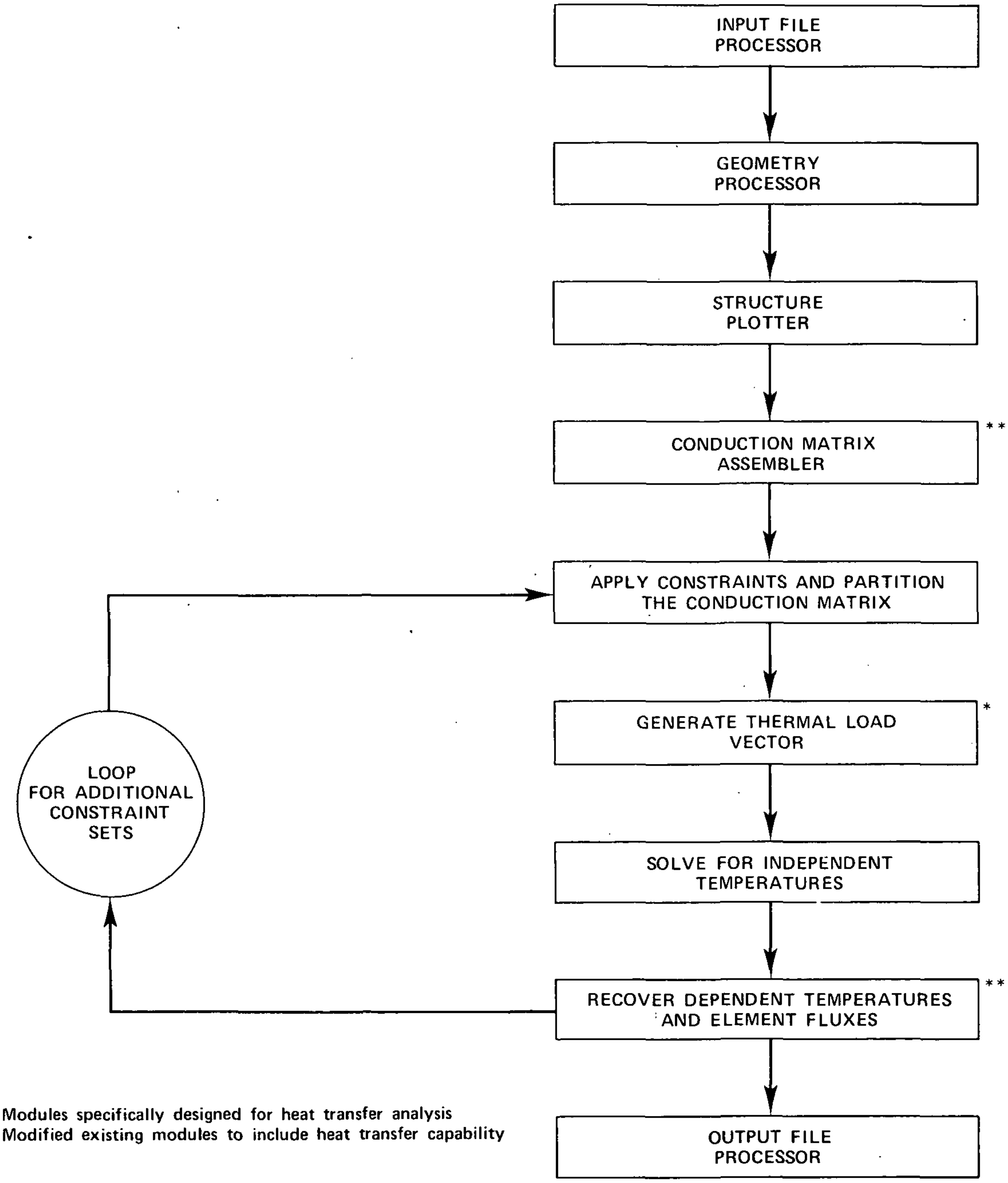
The Geometry Processor generates coordinate system transformation matrices, tables of grid point locations, a table defining the heat conduction elements connected to each grid point, and other miscellaneous tables such as those defining static thermal loads at grid points.

The Structure Plotter generates tape output for an automatic plotter that will plot the structure as formed by heat conduction elements (i.e., the location of grid points and the boundaries of elements) in one of several available three-dimensional projections. The structure plotter is particularly useful for the detection of errors in grid point coordinates and in the connection of elements to grid points.

The Conduction Matrix Assembler generates heat conduction and convection matrices referred to the grid points from tabular information generated by the Input File Processor and the Geometry Processor.

In the next block, the conduction matrix is reduced to the form in which it is finally solved through the imposition of single and multi-point constraints, and the use of matrix partitioning (optional).

Thermal load vectors are then generated from a variety of sources (concentrated thermal loads at grid points, uniform heat fluxes on surfaces, and prescribed temperatures) and are reduced to final form by the application of constraints and matrix partitioning.



*Modules specifically designed for heat transfer analysis
**Modified existing modules to include heat transfer capability

Figure 2.9. Simplified flow diagram for linear steady-state thermal analysis.

The solution for independent temperatures is accomplished in two steps: Decomposition of the conduction matrix [K] into upper and lower triangular factors; and solution for $\{u\}$ for specific thermal load vectors, $\{P\}$, by means of successive substitution into the equations represented by the triangular factors of [K] (the so-called forward and backward passes). All thermal load vectors are processed before proceeding to the next functional block.

In the eighth block of figure 2.9, dependent temperatures are determined from the independent temperatures by means of the conditions of constraint. The heat flow rates and gradients in each element are then computed from knowledge of the temperatures at the vertices of the elements and the intrinsic equations of the element. Finally the Output File Processor prepares the results of the analysis for printing.

The Loop for Additional Constraint Sets shown in figure 2.9, is introduced to facilitate solutions for different boundary conditions, which are applied by means of single point constraints.

The solution of the equation representing linear steady-state thermal analysis of the form

$$[K]\,\{u\} = \{P\} \tag{6-1}$$

is accomplished using the results of the decomposition procedure described in section 2.2 of the NASTRAN Theoretical Manual. Replacing [K] by its triangular factors, equation (6-1) becomes

$$[L]\,[U]\,\{u\} = \{P\} \tag{6-2}$$

where [L] is a lower unit triangle and [U] is an upper triangle.

Define

$$\{y\} = [U]\,\{u\} \tag{6-3}$$

Then substituting into equation (6-2)

$$[L]\,\{y\} = \{P\} \tag{6-4}$$

The solution of equation (6-4) for $\{y\}$ is called the forward pass, and the subsequent solution of equation (6-3) for $\{u\}$ is called the backward pass.

In the solution algorithm, y_1 is evaluated from the leading element of [L], and the first column of [L] is multiplied by y_1 and transferred to the right-hand side of equation (6-4). The procedure is repeated for the second and succeeding columns of [L] until all elements of $\{y\}$ have been evaluated. The algorithm for obtaining $\{u\}$ is similar except that the columns of [U] are required in reverse order. Multiple $\{P\}$ vectors can be handled simultaneously up to the limit of the working space available in main memory. The same general procedures are used for both symmetric and asymmetric matrices.

The forward pass requires the reading of both the right-hand vectors and the lower triangular factor from secondary storage devices. Each term of the lower triangular factor is used only once; thus if there are a small number of right-hand vectors, the computing time for the forward pass is dominated by the time required to read the lower triangular factor from secondary storage.

The backward pass is accomplished in two steps. First, the upper triangular factor is read backward and written forward on a separate file so that the last column of [U] appears first. In the case of symmetric matrices, the actual file read backward is the lower triangular factor, and the file written forward is renamed as the upper triangular factor. This is part of the triangular decomposition routine and takes place immediately after the completion of the decomposition. The reason for this preliminary operation is that all NASTRAN files are sequential, and since it takes several times as long to read a sequential file backward as forward, it is desirable to execute the backward read only once, even though the triangular factors may be used several times. This allows substantial time savings when making restarts and when multiple right-hand vectors cannot be held in main memory. The second step of the backward pass consists of solving equation (6-3) for u. It is made a part of the equation solution routine and, as in the case of the forward pass, the computer time often is dominated by the time required to read the upper triangular factor from secondary storage.

Following the determination of the solution vectors, a residual vector is determined for each solution vector as follows

$$\{\delta P\} = \{P\} - [K]\{u\} \quad . \tag{6-5}$$

The residual vector is used to calculate the following error ratio which is printed with the output.

$$\epsilon = \frac{\{u\}^T \{\delta P\}}{\{u\}^T \{P\}} \tag{6-6}$$

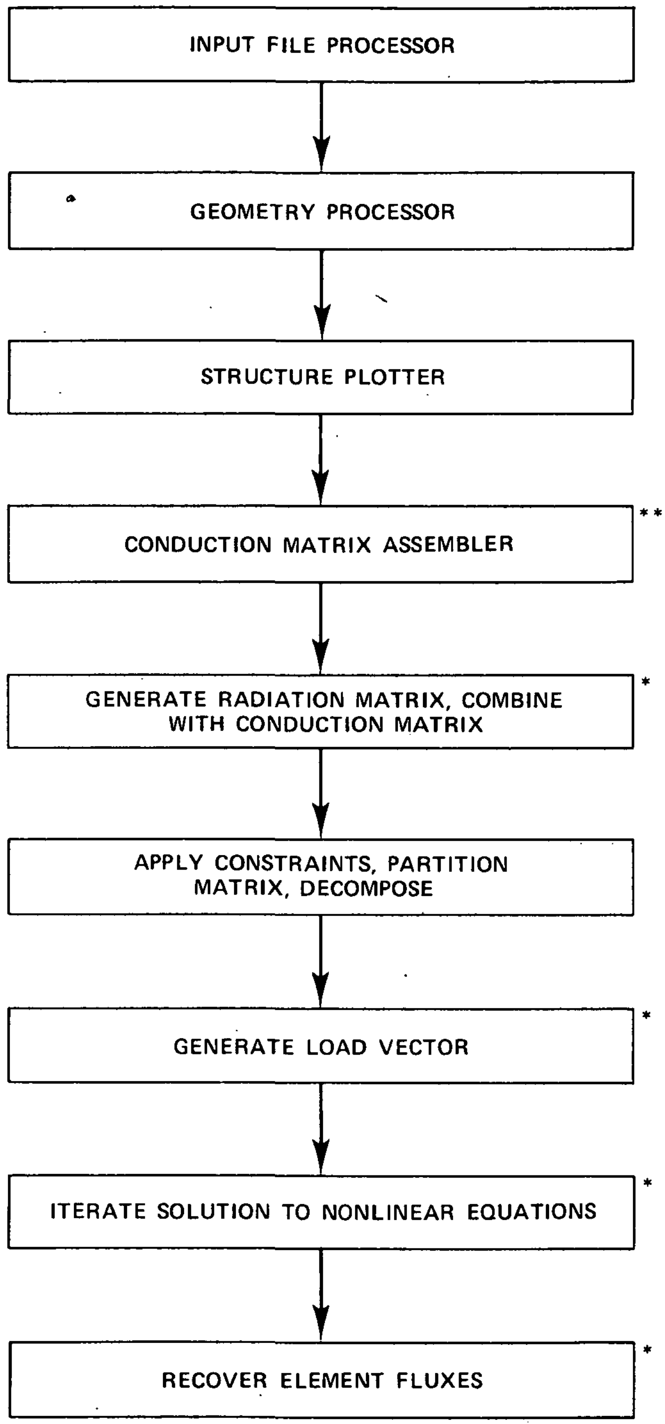
The magnitude of this error ratio gives an indication of the numerical accuracy of the solution vectors. The computer time required to calculate this error ratio is only a small fraction of the time required to determine the solution vector.

2.6.2 Nonlinear Steady-State Thermal Analysis

Nonlinear steady-state thermal analysis uses the rigid format (APP HEAT, SØL 3). The flow diagram is given in figure 2.10. The nonlinear properties permitted in steady-state heat transfer analysis with the NTA are radiation, temperature-dependent conductivity, and temperature-dependent convective film coefficient. The general form of the equation to be solved is

$$[K_{gg}]\{u_g\} + [R_{gg}]\{(u_g + T_a)^4\} = \{q_g\} + \{P_g\} \tag{6-7}*$$

*If $\{T\}$ is a vector, $\{T^4\}$ is defined as the vector whose components are the fourth power of the elements of $\{T\}$.



* Modules specifically designed for heat transfer analysis
** Modified existing modules to include heat transfer capability

Figure 2.10. Simplified flow diagram for nonlinear steady-state thermal analysis.

The temperature set $\{u_g\}$ includes temperature variables that are restrained by single point and multi-point constraints. The vector $\{q_g\}$ represents the powers required to sustain the prescribed temperatures. Subsections 2.5.1 and 2.5.2 describe the manner in which the heat conduction matrix, $[K_{gg}]$, the radiation matrix, $[R_{gg}]$, and the applied heat flow vector, $\{P_g\}$, are formed from the properties of heat conduction elements and boundary surface elements. The elements of $[K_{gg}]$ may be functions of temperature.

The first step in the solution is to rewrite equation (6–7) in terms of the set of temperatures, $\{u_n\}$, from which multi-point constraints have been removed. The procedures used are identical to those described in section 3.5 of the NASTRAN Theoretical Manual for structural analysis. In order to avoid difficulties in interpolating temperatures to form the nonlinear terms, a restriction is placed on the form of the multi-point constraint relationships, namely that, if a grid point is attached to a heat conduction or boundary surface element with nonlinear properties, the constraint relationship is restricted to be an "equivalence." The term "equivalence" means that the constrained temperature is set equal to a specified independent temperature.

The form of the thermal equilibrium equation after the multi-point dependent temperatures $\{u_m\}$ have been eliminated is

$$[K_{nn}]\{u_n\} + [R_{nn}]\{(u_n + T_a)^4\} = \{q_n\} + \{P_n\} \tag{6-8}$$

If $\{u_n\}$ is partitioned into $\{u_f\}$ (free points) and $\{u_s\}$ (Single point constraints), equation (6–8) may be written in partitioned form

$$\left[\begin{array}{c|c} K_{ff} & K_{fs} \\ \hline K_{sf} & K_{ss} \end{array}\right]\left\{\begin{array}{c} u_f \\ \hline u_s \end{array}\right\} + \left[\begin{array}{c|c} R_{ff} & R_{fs} \\ \hline R_{sf} & R_{ss} \end{array}\right]\left\{\left(\begin{array}{cc} u_f + & T_a \\ \hline u_s & T_a \end{array}\right)^4\right\} = \left\{\begin{array}{c} 0 \\ \hline q_s \end{array}\right\} + \left\{\begin{array}{c} P_f \\ \hline P_s \end{array}\right\} \tag{6-9}$$

The components of $\{u_s\}$ have values prescribed by the user, and the lower half of equation (6–9) is used to evaluate the single point "powers" of constraint $\{q_s\}$ during data recovery. Rearranging the top half of equation (6–9) we obtain

$$[K_{ff}]\{u_f\} + [R_{ff}]\{(u_f + T_a)^4\} = \{P_f\} - [K_{fs}]\{u_s\} - [R_{fs}]\{(u_s + T_a)^4\} \tag{6-10}$$

Equation (6-10) is solved by an iterative method. The technique used is to expand $\{u_f\}$ into constant, linear, and higher order terms with respect to an initial estimate, $\{u_f^1\}$, supplied by the user. The linear terms are kept on the left-hand side of equation (6-10) and all other terms are placed on the right-hand side, where they are evaluated precisely for the current estimate of $\{u_f\}$. If we define $\{L\}$ to be the left-hand side of equation (6-10), then the new left-hand side is

$$\{L^*\} = \left[\frac{\partial L}{\partial u_f}\right]\{u_f\} = [K_{ff}^1]\{u_f\} + 4[R_{ff}]\,[(u_f^1 + T_a)^3]\{u_f\} \tag{6-11}$$

where the partial derivatives are evaluated for $\{u_f\} = \{u_f^1\}$. Using this expression, equation (6-10) may be written as

$$[K_{ff}^*]\{u_f\} = [K_{ff}^1 - K_{ff}]\{u_f\} + [R_{ff}]\,(4\lceil(u_f^1 + T_a)^3\rfloor\{u_f\} - \{(u_f + T_a)^4\})$$

$$+\{P_f\} - [K_{fs}]\{u_s\} - [R_{fs}]\,\{(u_s + T_a)^4\} \qquad (6\text{-}12)$$

where

$$[K_{ff}^*] = [K_{ff}^1] + 4[R_{ff}]\,\lceil(u_f^1 + T_a)^3\rfloor \qquad (6\text{-}13)$$

It is convenient, for computational purposes, to combine terms proportional to $\{u_f\}$ and $\{u_s\}$ on the right-hand side of equation (6-12) to produce terms proportional to $\{u_n\}$. Thus if we define

$$\{u_n^1\} = \begin{Bmatrix} u_f^1 \\ \hline u_s \end{Bmatrix}$$

$$\begin{aligned}
[K_{fs}^*] &= [K_{fs}^1] + 4[R_{fs}]\,\lceil(u_s + T_a)^3\rfloor \\
[K_{fn}] &= [K_{ff} \mid K_{fs}] \\
[K_{fn}^1] &= [K_{ff}^1 \mid K_{fs}^1] \\
[R_{fn}] &= [R_{ff} \mid R_{fs}]
\end{aligned} \qquad (6\text{-}14)$$

then equation (6-12) may be written as

$$[K_{ff}^*]\,\{u_f\} = \{N_f\} \qquad (6\text{-}15)$$

where

$$\{N_f\} = \{P_f - K_{fs}^* u_s\} - [K_{fn} - K_{fn}^1]\,\{u_n\} - [R_{fn}](\{(u_n + T_a)^4\} - 4\lceil(u_n^1 + T_a)^3\rfloor\{u_n\}) \qquad (6\text{-}16)$$

The first term in equation (6-16) is a constant, and the other terms are functions of temperature. Equation (6-15) is an exact relationship. The iteration algorithm consists of evaluating $\{N_f\}$ for $\{u_n\} = \{u_n^i\}$, the current estimate of the temperature distribution, and of solving equation (6-15) to obtain a new estimate, $\{u_f^{i+1}\}$, of the unknown temperatures. The starting vector is $\{u_f^1\}$, supplied by the user.

The algorithm is simple enough, but the number of iterations to obtain satisfactory convergence (if indeed convergence can be achieved) remains an open question. The question of convergence can be treated without difficulty in a small neighborhood of the correct solution within which the nonlinear load may be approximated as a linear function of the error in the temperature distribution. The iteration algorithm is

$$[K^*_{ff}]\,\{u_f^{i+1}\} = \{N_f^i\} \tag{6-17}$$

As an approximation, let

$$\{N_f^i\} = \{N_f\} + [C]\,\{u_f^i - u_f\} = [N_f] + [C]\,\{\delta u^i\} \tag{6-18}$$

where $[C]$ is the matrix of the partial derivatives of $\{N_f\}$ with respect to changes in $\{u_f\}$. Substituting equation (6–18) into equation (6–17) and using equation (6–15), the iteration algorithm is, approximately,

$$[K^*_{ff}]\{\delta u^{i+1}\} = [C]\{\delta u^i\} \tag{6-19}$$

Equation (6-19) resembles the power method of eigenvalue extraction (see section 10.4 of the NASTRAN Theoretical Manual) and its convergence is related to the distribution of the eigenvalues of the associated eigenvalue problem.

$$[K^*_{ff} - \lambda C]\{\delta u\} = \{0\} \tag{6-20}$$

In order to establish the condition for convergence, expand the iterates $\{\delta u^i\}$ and $\{\delta u^{i+1}\}$ in terms of the eigenvectors $\{\phi_j\}$, i.e.,

$$\{\delta u^i\} = \sum_j \alpha_j^i \{\phi_j\} \tag{6-21}$$

$$\{\delta u^{i-1}\} = \sum_j \alpha_j^{i-1} \{\phi_j\} \tag{6-22}$$

It can be proved quite generally (see section 10.4.4.3 of the NASTRAN Theoretical Manual) that a property of the linearized iteration algorithm is that

$$\alpha_j^{i-1} = \lambda_j\, \alpha_j^i \tag{6-23}$$

where λ_j is the eigenvector corresponding to $\{\phi_j\}$. Thus, it is seen that, if $|\lambda_j| < 1$, α_j^i will increase in magnitude at each iteration and the algorithm will be divergent. The necessary and sufficient condition for convergence in a small neighborhood of the correct solution is that all eigenvalues of equation (6–20) have magnitudes greater than one.

NASTRAN provides both an estimate of the lowest eigenvalue and an estimate of the error in the solution after each iteration. If the iteration has proceeded to the point where one eigenvector, $\{\phi_1\}$, dominates the solution, it is seen from equations (6–21), (6–22) and (6–23) that

$$\{\delta u^i - \delta u^{i-1}\} = \frac{1}{\lambda_1} \{\delta u^{i-1} - \delta u^{i-2}\} \tag{6-24}$$

so that the ratios of successive increments in the elements of the solution vector provide an estimate of the lowest eigenvalue. By analogy with a procedure used in the inverse power method with shifts (see section 10.4 of the NASTRAN Theoretical Manual) a single weighted estimate is obtained by multiplying both sides of equation (6–24) by the transpose of the nonlinear load vector. Thus, the estimate is

$$\lambda_1^i = \frac{\{N_f^i\}^T \{u_f^{i-1} - u_f^{i-2}\}}{\{N_f^i\}^T \{u_f^i - u_f^{i-1}\}} \tag{6-25}$$

Equation (6–25) is evaluated after every iteration starting with the third, i = 4.

The vector $\{\delta u^i\}$ is the error in the solution at the beginning of the ith iteration. In order to obtain a measure of the error, we observe, from equations (6–21), (6–22) and (6–23) that if only one eigenvector is present

$$\{\delta u^i - \delta u^{i-1}\} = (1 - \lambda_1)\{\delta u^i\} \tag{6-26}$$

The measure of the error in temperature used in the $\overline{\text{N}}$TA is the ratio of the work done by the nonlinear loads acting on the error vector to the work done by the nonlinear loads acting on the total solution, i.e.,

$$\epsilon_T^i = \left| \frac{\{N^i\}^T \{\delta u^i\}}{\{N^i\}^T \{u_f\}} \right| = \left| \frac{1}{(\lambda_1^i - 1)} \frac{\{N_f^i\}^T \{u_f^i - u_f^{i-1}\}}{\{N_f^i\}^T \{u_f^i\}} \right| \tag{6-27}$$

Another error measure is also provided, which measures the error in the applied heat flux, including nonlinear terms. That measure is

$$\epsilon_p^i = \frac{||N_f^i - [K_{ff}^*]\, u_f^i||}{||N_f^1||} = \frac{||N_f^i - N_f^{i-1}||}{||N_f^1||} \tag{6-28}$$

where $||X||$ is the Euclidean norm of the vector $\{X\}$.

The iteration algorithm will terminate for any of the following reasons:

1. ϵ_T^i is less than a user-specified value and also $|\lambda_1^i| > 1$: Normal convergence.
2. $|\lambda_1^i| < 1$ for $i \geq 4$: The algorithm is assumed to be divergent.
3. The number of iterations reaches the maximum number specified by the user.
4. The available time is used up.

In all cases, the values of ϵ_T^i, ϵ_p^i and λ_1^i may be output for every iteration, and the solution vector for the last iteration will be output.

Radiated heat flux is proportional to the fourth power of the temperature, thereby providing a very strong nonlinear effect if the radiation terms are large compared to other terms. In order to gauge the effect of radiation on convergence of the iteration algorithm, consider an isolated perfectly conducting body in thermal equilibrium with radiant input from distant sources. The thermal equilibrium equation is

$$Ru^4 = P \tag{6-29}$$

where u is measured on an absolute scale, and P is constant. The user supplies an estimate of the temperature, u_1. The iteration algorithm is, in accordance with the preceding discussion,

$$(4Ru_1^3)u_{i+1} = P - R[(u_i)^4 - 4u_1^3 u_i] \tag{6-30}$$

The derivative of the right-hand side at the correct solution ($u_i = u$) is

$$C = -4R(u_1^3 - u^3) \tag{6-31}$$

so that the eigenvalue problem corresponding to equation (6–20) is

$$[4Ru_1^3 - \lambda 4R(u_1^3 - u^3)] \ \delta u = 0 \tag{6-32}$$

The eigenvalue is

$$\lambda = \frac{u_1^3}{u_1^3 - u^3} \tag{6-33}$$

The critical value for convergence, $\lambda = -1$, is achieved if

$$u_1^3 = \frac{1}{2}u^3$$

or

$$u_1 = 0.794u$$

Thus, the solution converges if u_1 is greater than about 80 percent of the correct temperature, measured on an absolute scale. The user can assure convergence, at the expense of extra iterations, by overestimating the temperature.

2.6.3 Transient Thermal Analysis

Transient thermal analysis permits both linear and nonlinear options using the same rigid format (APP HEAT, SØL 9). The nonlinear terms permitted in transient thermal analysis include radiation and the general purpose nonlinear elements described in section 11.2 of the NASTRAN User's Manual. However, nonlinearities arising from temperature-dependent thermal conductivity, convective film coefficient, and heat capacity are not permitted. This is due to the fact that the computational effort required to recalculate the thermal conduction and heat capacitance matrices at each time step by the finite element method used in the NASTRAN Thermal Analyzer was judged to be excessive. The general purpose nonlinear elements can, however, be applied to model nonlinear convective coupling as well as other relatively simple nonlinear relationships. The simplified flow diagrams for transient thermal analysis is shown in figure 2.11.

The general equation solved in transient analysis is of the form

$$[B]\{\dot{u}\} + [K]\{u\} = \{P\} + \{N\} \tag{6-34}$$

The thermal conduction matrix [K] includes linearized radiation terms. It is, in fact, identical to $[K^*_{ff}]$ given by equation (6–13) of the preceding subsection 2.6.2, except possibly for terms due to "extra points," (see section 9.3.2 of the NASTRAN Theoretical Manual). The nonlinear term in equation (6–34) is

$$\{N\} = \{N^e\} + [R]\,(4[(u^1 + T_a)^3]\{u\} - \{(u + T_a)^4\}) \tag{6-35}$$

where $\{N^e\}$ is attributed to general purpose nonlinear elements and the second term is attributed to radiative exchanges. An option is available to substitute $\{u^1\}$ for $\{u\}$ in the second term, which reduces it to a constant vector and which, thereby, linearizes the effect of radiation.

The thermal load vector $\{P\}$ may be formed in the same manner as for the steady-state heat transfer analysis with certain parameters permitted to be functions of time. These include the prescribed volumetric (internally generated heat) and surface heat fluxes, and the prescribed vector heat flux. In the latter case, both the direction and the magnitude of the heat flux are permitted to be functions of time. Prescribed temperatures at grid points, and the ambient temperatures used for convective heat transfer are treated in the same manner as prescribed displacements in dynamic analysis. It requires the user to connect a large scalar conduction element, K_0, to the grid point in question and also to apply a thermal load $P = TK_0$ to the grid point where T is the desired temperature function of time.

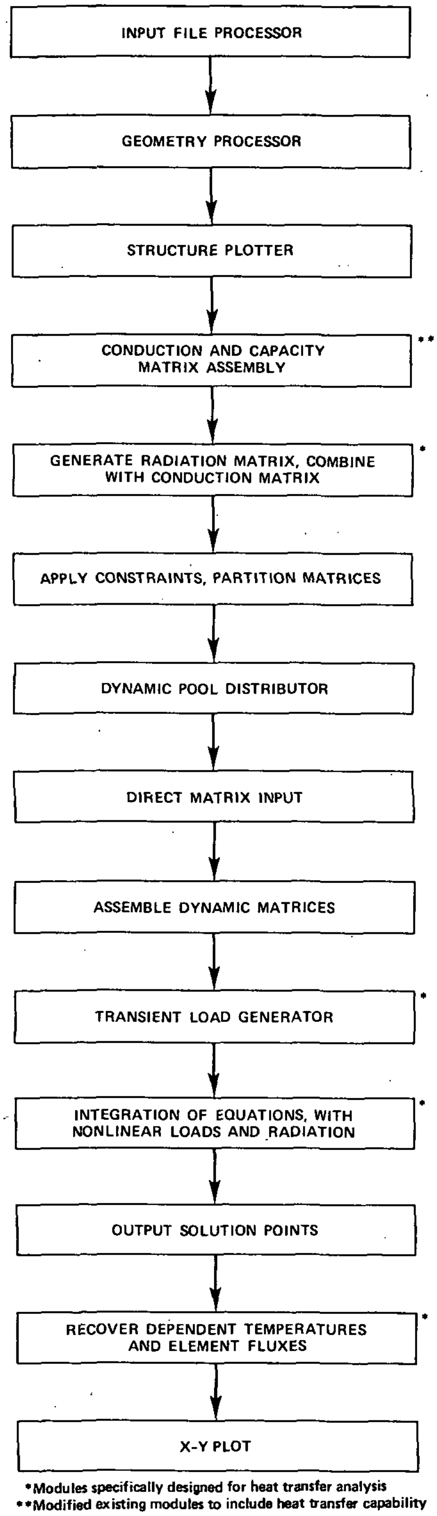


Figure 2.11. Simplified flow diagram for transient thermal analysis.

The algorithm used to integrate equation (6–34) has been selected with the following criteria in mind:

1. Unconditional stability for linear problems, regardless of the size of the time step
2. Ability to handle a singular heat capacity matrix
3. Good stability for nonlinear problems
4. Good efficiency
5. High accuracy

A useful general observation is that stability, efficiency and accuracy are conflicting requirements that must be compromised. The algorithm that has been selected can satisfy the first two criteria and scores reasonably well on the last three. Basically, it is a difference equation approximation to equation (6–34) with a free parameter that is adjusted to produce a compromise between the stability, efficiency and accuracy requirements. In this respect it is analogous to the Newmark β method used in structural dynamics (see section 11.3 of the NASTRAN Theoretical Manual). The form of the difference equation is

$$[K]\{\beta u_{n+1} + (1-\beta) u_n\} + \frac{1}{\Delta t}[B]\{u_{n+1} - u_n\} =$$

$$\{\beta P_{n+1} + (1-\beta) P_n\} + (1+\beta)\{N_n\} - \beta\{N_{n-1}\} \qquad (6\text{–}36)$$

The subscript n refers to the nth time step. The parameter, β, may be selected by the user in the range $0 < \beta < 1$. Putting terms proportional to $\{u_{n+1}\}$ on the left side yields the iteration algorithm

$$\left[\frac{1}{\Delta t} B + \beta K\right]\{u_{n+1}\} = \left[\frac{1}{\Delta t} B - (1-\beta) K\right]\{u_n\}$$

$$+ \beta\{P_{n+1}\} + (1-\beta)\{P_n\} + (1+\beta)\{N_n\} - \beta\{N_{n-1}\} \qquad (6\text{–}37)$$

The matrix $[B/\Delta t + \beta K]$ is first decomposed into its triangular factors from which the equations are solved at each time step using a forward and backward substitution procedure (see section 2.3 of the NASTRAN Theoretical Manual). The time step, Δt, may be changed at discrete times by the user. Certain values of the parameter β result in well-known algorithms, viz.,

$\beta = 0$: Euler integration
$\beta = 1/2$: Central differences
$\beta = 1$: Backward differences

Euler integration ($\beta = 0$) is usually the most efficient choice because only the [B] matrix, which is frequently diagonal, is decomposed. However, Euler integration cannot be used if [B] is singular and it suffers with respect to both stability and accuracy as will be seen.

The effect of β on stability will be examined for the linear case, for which the matrix equation of motion is

$$[B]\left\{\dot{u}\right\} + [K]\left\{u\right\} = P \tag{6-38}$$

A more convenient set of equations is obtained by a transformation of $\{u\}$ into modal coordinates, $\{\xi_i\}$:

$$\left\{u\right\} = [\phi]\left\{\xi_i\right\} \tag{6-39}$$

where each column of $[\phi]$ is an eigenvector of equation (6–38). The equation for each modal coordinate is uncoupled from the others and has the form

$$\dot{\xi}_i + \lambda_i \xi_i = P_i \tag{6-40}$$

where λ_i is the eigenvector and P_i is the generalized force on ξ_i. The system of equations is stable if all $\lambda_i \geqslant 0$.

Applying the integration algorithm to equation (6–40) we obtain

$$\frac{1}{\Delta t}(\xi_{n+1} - \xi_n) + \lambda_i(\beta\,\xi_{n+1} + (1-\beta)\,\xi_n) = \beta\,P_{n+1} + (1-\beta)P_n \tag{6-41}$$

where the subscript (i) has been omitted for clarity. The solution for the homogeneous case ($P_n = P_{n+1} = 0$) has the property that

$$\xi_{n+1} = E\xi_n \tag{6-42}$$

where E is a constant, called the shift operator. If $|E| \leqslant 1$, the homogeneous solution is stable because it approaches zero for large n. By substituting equation (6–42) into equation (6–41) for the homogeneous case, we obtain

$$\left[\frac{1}{\Delta t}(E-1) + \lambda_i(\beta\,E + 1 - \beta)\right]\xi_n = 0 \tag{6-43}$$

Setting the coefficient of ξ_n to zero, which must occur if ξ_n is not to be zero, produces a functional relationship between E, β, and $\lambda_i\Delta t$, which may be expressed in the form

$$\lambda_i \Delta t = \frac{1 - E}{E\,\beta + 1 - \beta} \tag{6-44}$$

The range of E for stability is $-1 \leqslant E \leqslant 1$. Substitution of the upper limit into equation (6–44) is seen to produce no restriction on the time step. Substitution of the lower limit, however, gives as a stability limit

$$\lambda_i \Delta t = \frac{2}{1 - 2\beta} \tag{6-45}$$

Thus, if $\beta = 0$ (Euler integration) the stability limit is $\lambda_i \Delta t = 2$. Since λ_i is the reciprocal of the time constant of the ith mode of the system, the practical restriction on Euler integration is that the time step can be no greater than twice the smallest decay time constant of the system. If $\beta = 0.5$, there is seen to be no limit on the time step, nor is there for $\beta > 0.5$, which can most readily be seen by solving equation (6–44) for E, i.e.,

$$E = \frac{1 - (1 - \beta)\,\lambda_i \Delta t}{1 + \beta\,\lambda_i \Delta t} \tag{6-46}$$

From the viewpoint of stability then, β should be chosen greater than or equal to 0.5. For linear problems $\beta = 0.5$ is adequate, but for nonlinear problems in which the nonlinear terms must necessarily be evaluated at the nth and earlier time steps, a larger value of β may be advisable.

Insight into the question of accuracy can be gained by examining the eigenvalues produced by the integration algorithm and comparing them with the eigenvalues of the real system. The eigenvalue, Λ_i, produced by numerical integration is defined implicitly by

$$\xi_{n+1} = e^{-\Lambda_i \Delta t}\,\xi_n \tag{6-47}$$

or, by comparison with equation (6–42)

$$\Lambda_i = \frac{-1}{\Delta t}\,\ell n\,E \tag{6-48}$$

so that, using equation (6–46)

$$\Lambda_i = \frac{-1}{\Delta t}\,\ell n\left(\frac{1 - (1 - \beta)\,\lambda_i \Delta t}{1 + \beta\,\lambda_i \Delta t}\right) \tag{6-49}$$

If $\lambda_i \Delta t$ is assumed to be less than one, equation (6–49) can be evaluated by power series expansion with the result

$$\Lambda_i = \lambda_i\left[1 + \left(\beta - \frac{1}{2}\right)(\lambda_i \Delta t) - \left(\frac{1}{12} + \left(\beta - \frac{1}{2}\right)^2\right)(\lambda_i \Delta t)^2 + \cdots\right] \tag{6-50}$$

It is seen that, if the time step, Δt, is small compared to the decay time constant of the mode, $1/\lambda_i$, the error will be a minimum near $\beta = 0.5$. Since efficiency or stability considerations will be overriding in many cases, the choice of β is left to the user. The default value, in the event that the user declines to make a choice, is $\beta = 0.55$.

The provisions for initial conditions are as follows. The initial thermal load (for equation (6-37) at n = 0) is taken as

$$\{P_0\} = [K]\{u_0\} - \{N_0\} \tag{6-51}$$

which sets $\{\dot{u}\}$ to zero initially (see equation (6-34)). Since $\{u_n\}$ is not defined for negative n, the nonlinear load at $t = -\Delta t$ is taken to be

$$\{N_{-1}\} = \{N_0\} \tag{6-52}$$

Equations (6-51) and (6-52) have the property that they yield smooth results when step loads are applied to degrees of freedom without thermal capacity. Special conditions are also needed if it is desired to change the time step. The situation is similar to the starting equations except that the new initial velocity vector, $\{\dot{u}\}$, is set equal to the old final vector. Let N be the index of the last step with the previous time step Δt_1. Let Δt_2 be the new time step and let the time step counter be reset to zero. The new initial conditions are

$$\{u_0\} = \{u_N\} \tag{6-53}$$

$$\{\dot{u}_0\} = \frac{1}{\Delta t_1}\{u_N - u_{N-1}\} \tag{6-54}$$

The new initial thermal load is

$$\{P_0\} = [K]\{u_0\} - \{N_0\} - [B]\{\dot{u}_0\} \tag{6-55}$$

Interpolation is used to produce an estimate of the nonlinear load at $t = -\Delta t_2$

$$\{N_{-1}\} = \frac{\Delta t_2}{\Delta t_1}\{N_{N-1}\} + \left(1 - \frac{\Delta t_2}{\Delta t_1}\right)\{N_N\} \tag{6-56}$$

These provisions are designed to minimize discontinuities associated with time step changes. The coefficient matrices in equation (6-37) are recomputed, and the matrix coefficient of $\{u_{n+1}\}$ is decomposed before continuing the integration with the new initial values.

2.7 Rigid Format DMAP Listings

The NTA computer program will perform mathematical calculations according to a sequence of module calls either built into the program or generated by the user. The former solution capability is known as the Rigid Format and the great majority of problems will be solved via this method. The latter capability is provided in order to allow the advanced user (using the program's matrix routines, which are available for general use) to solve problems with features not accounted for in any of the rigid formats. There are two options available to the user in this latter capability: (1) ALTER allows modifications of a rigid format. Typical uses of the ALTER feature are to schedule an exit at an intermediate point in a solution for the purpose of checking intermediate output, to schedule the printing of a table or a matrix for diagnostic purposes, and to add or delete a functional module from the sequence of operating instructions, (2) DMAP (Direct Matrix Abstraction Program) allows the user to write his own sequence of executive instructions. DMAP is a user-oriented programming language of macro instructions which, like FORTRAN, has many rules which must be followed to be interpretable by NASTRAN. DMAP is also used in the construction of rigid formats, which differ from user-generated sequences mainly in that restart tables are provided. For further information, see section 5 of the NASTRAN User's Manual.

While information concerning the coding details of individual functional modules may be found in the NASTRAN Programmer's Manual,[5] the DMAP listings pertaining to the three solution algorithms are given on the following pages for those users who may wish to modify the rigid formats to satisfy their specific needs.

2.7.1 DMAP Sequence for Linear Steady-State Thermal Analysis

```
RIGID FORMAT DMAP LISTING
SERIES M1

RIGID FORMAT 1

   N A S T R A N   S O U R C E   P R O G R A M   C O M P I L A T I O N
DMAP-DMAP INSTRUCTION
NO.

  1  BEGIN     NO.1 STATIC ANALYSIS - SERIES M1 $

  2  FILE      LLL=TAPE $

  3  FILE      QG=APPEND/PGG=APPEND/UGV=APPEND/GM=SAVE/KNN=SAVE $

  4  GP1       GEOM1,GEOM2,/GPL,EQEXIN,GPDT,CSTM,BGPDT,SIL/V,N,LUSET/  C,N,
               123/V,N,NOGPDT $

  5  SAVE      LUSET $

  6  CHKPNT    GPL,EQEXIN,GPDT,CSTM,BGPDT,SIL $

  7  GP2       GEOM2,EQEXIN/ECT $

  8  CHKPNT    ECT $

  9  PLTSET    PCDB,EQEXIN,ECT/PLTSETX,PLTPAR,GPSETS,ELSETS/V,N,NSIL/   V,N,
               JUMPPLOT $

 10  SAVE      NSIL,JUMPPLOT $

 11  PRTMSG    PLTSETX// $

 12  SETVAL    //V,N,PLTFLG/C,N,1/V,N,PFILE/C,N,0 $

 13  SAVE      PLTFLG,PFILE $

 14  COND      P1,JUMPPLOT $

 15  PLOT      PLTPAR,GPSETS,ELSETS,CASECC,BGPDT,EQEXIN,SIL,,/PLOTX1/ V,N,
               NSIL/V,N,LUSET/V,N,JUMPPLOT/V,N,PLTFLG/V,N,PFILE $

 16  SAVE      JUMPPLOT,PLTFLG,PFILE $

 17  PRTMSG    PLOTX1// $

 18  LABEL     P1 $

 19  CHKPNT    PLTPAR,GPSETS,ELSETS $

 20  GP3       GEOM3,EQEXIN,GEOM2/SLT,GPTT/C,N,123/V,N,NOGRAV/C,N,123 $

 21  SAVE      NOGRAV $

 22  PARAM     //C,N,AND/V,N,SKPMGG/V,N,NOGRAV/V,Y,GRDPNT $

 23  PURGE     MGG/SKPMGG $

 24  CHKPNT    SLT,GPTT,MGG $
```

RIGID FORMAT DMAP LISTING
SERIES M1

RIGID FORMAT 1

```
   N A S T R A N   S O U R C E   P R O G R A M   C O M P I L A T I O N
DMAP-DMAP INSTRUCTION
 NO.

 25  TA1,      ,ECT,EPT,BGPDT,SIL,GPTT,CSTM/EST,,GEI,ECPT,GPCT/V,N,LUSET/ C,N,
               123/V,N,NOSIMP/C,N,0/V,N,NOGENL/V,N,GENEL $

 26  SAVE      NOSIMP,NOGENL,GENEL $

 27  PARAM     //C,N,AND/V,N,NOELMT/V,N,NOGENL/V,N,NOSIMP $

 28  COND      ERROR4,NOELMT $

 29  PURGE     GPST/NOSIMP/OGPST/GENEL $

 30  CHKPNT    EST,ECPT,GPCT,GEI,GPST,OGPST $

 31  COND      LBL1,NOSIMP $

 32  SMA1      CSTM,MPT,ECPT,GPCT, DIT/KGGX,,GPST/V,N,NOGENL/V,N,NOK4GG/  V,Y,
               OPTION $

 33  CHKPNT    GPST,KGGX $

 34  COND      LBL1,SKPMGG $

 35  SMA2      CSTM,MPT,ECPT,GPCT,DIT/MGG,/V,Y,WTMASS=1.0/V,N,NOMGG/V,N,NOBGG/
               V,Y,COUPMASS/V,Y,CPBAR/V,Y,CPROD/V,Y,CPQUAD1/V,Y,CPQUAD2/   V,
               Y,CPTRIA1/V,Y,CPTRIA2/V,Y,CPTUBE/V,Y,CPQDPLT/V,Y,CPTRPLT/ V,Y,
               CPTRBSC $

 36  SAVE      NOMGG $

 37  CHKPNT    MGG $

 38  COND      LBL1,GRDPNT $

 39  COND      ERROR2,NOMGG $

 40  GPWG      BGPDT,CSTM,EQEXIN,MGG/OGPWG/V,Y,GRDPNT=-1/V,Y,WTMASS $

 41  OFP       OGPWG,,,,,//V,N,CARDNO $

 42  SAVE      CARDNO $

 43  LABEL     LBL1 $

 44  EQUIV     KGGX,KGG/NOGENL $

 45  CHKPNT    KGG $

 46  COND      LBL11A,NOGENL $

 47  SMA3      GEI,KGGX/KGG/V,N,LUSET/V,N,NOGENL/V,N,NOSIMP $

 48  CHKPNT    KGG $
```

RIGID FORMAT DMAP LISTING
SERIES M1

RIGID FORMAT 1

```
   N A S T R A N   S O U R C E   P R O G R A M   C O M P I L A T I O N
DMAP-DMAP INSTRUCTION
 NO.

 49  LABEL    LBL11A $

 50  PARAM    //C,N,MPY/V,N,NSKIP/C,N,0/C,N,0 $

 51  JUMP     LBL11 $
                                                        Top of DMAP Loop
 52  LABEL    LBL11 $

 53  GP4      CASECC,GEOM4,EQEXIN,SIL,GPDT/RG,YS,USET,/V,N,LUSET/V,N,MPCF1/
              V,N,MPCF2/V,N,SINGLE/V,N,OMIT/V,N,REACT/V,N,NSKIP/V,N,REPEAT/
              V,N,NOSET/V,N,NOL/V,N,NOA $

 54  SAVE     MPCF1,MPCF2,SINGLE,OMIT,REACT,NSKIP,REPEAT,NOSET,NOL,NOA $

 55  COND     ERROR3,NOL $

 56  PARAM    //C,N,AND/V,N,NOSR/V,N,SINGLE/V,N,REACT $

 57  PURGE    KRR,KLR,QR,DM/REACT/GM/MPCF1/GO,KOO,LOO,UOO,PO,UOOV,RUOV/OMIT/
              PS,KFS,KSS/SINGLE/QG/NOSR $

 58  EQUIV    KGG,KNN/MPCF1 $

 59  CHKPNT   KRR,KLR,QR,DM,GM,GO,KOO,LOO,UOO,PO,UOOV,QG,PS,KFS,KSS,USET,RG,
              YS,RUOV,KNN $

 60  COND     LBL4,GENEL $

 61  GPSP     GPL,GPST,USET,SIL/OGPST $

 62  OFP      OGPST,,,,,//V,N,CARDNO $

 63  SAVE     CARDNO $

 64  LABEL    LBL4 $

 65  COND     LBL2,MPCF2 $

 66  MCE1     USET,RG/GM $

 67  CHKPNT   GM $

 68  MCE2     USET,GM,KGG,,,/KNN,,, $

 69  CHKPNT   KNN $

 70  LABEL    LBL2 $

 71  EQUIV    KNN,KFF/SINGLE $

 72  CHKPNT   KFF $
```

RIGID FORMAT DMAP LISTING
SERIES M1

RIGID FORMAT 1

N A S T R A N   S O U R C E   P R O G R A M   C O M P I L A T I O N

```
DMAP-DMAP INSTRUCTION
 NO.

 73  COND      LBL3,SINGLE $
 74  SCE1      USET,KAA,,,/KFF,KFS,KSS,,, $
 75  CHKPNT    KFS,KSS,KFF $
 76  LABEL     LBL3 $
 77  EQUIV     KFF,KAA/OMIT $
 78  CHKPNT    KAA $
 79  COND      LBL5,OMIT $
 80  SMP1      USET,KFF,,,/GO,KAA,KOO,LOO,UOO,,,,, $
 81  CHKPNT    GO,KAA,KOO,LOO,UOO $
 82  LABEL     LBL5 $
 83  EQUIV     KAA,KLL/REACT $
 84  CHKPNT    KLL $
 85  COND      LBL6,REACT $
 86  RBMG1     USET,KAA,/KLL,KLR,KRR,,, $
 87  CHKPNT    KLL,KLR,KRR $
 88  LABEL     LBL6 $
 89  RBMG2     KLL/LLL,ULL $
 90  CHKPNT    ULL,LLL $
 91  COND      LBL7,REACT $
 92  RBMG3     LLL,ULL,KLR,KRR/DM $
 93  CHKPNT    DM $
 94  LABEL     LBL7 $
 95  SSG1      SLT,BGPDT,CSTM,SIL,EST,MPT,GPTT,EDT,MGG,CASECC,DIT/PG/V,N,
               LUSET/V,N,NSKIP/V,Y,OPTION $
 96  CHKPNT    PG $
 97  EQUIV     PG,PL/NOSET $
 98  CHKPNT    PL $
```

RIGID FORMAT DMAP LISTING
SERIES M1

RIGID FORMAT 1

```
   N A S T R A N   S O U R C E   P R O G R A M   C O M P I L A T I O N
DMAP-DMAP INSTRUCTION
 NO.

 99  COND      LBL10,NOSET $

100  SSG2      USET,GM,YS,KFS,GO,DM,PG/QR,PO,PS,PL $

101  CHKPNT    QR,PO,PS,PL $

102  LABEL     LBL10 $

103  SSG3      LLL,ULL,KLL,PL,LOO,UOO,KOO,PO/ULV,UOOV,RULV,RUOV/V,N,OMIT/ V,Y,
               IRES=-1/V,N,NSKIP/V,N,EPSI $

104  SAVE      EPSI $

105  CHKPNT    ULV,UOOV,RULV,RUOV $

106  COND      LBL9,IRES $

107  MATGPR    GPL,USET,SIL,RULV//C,N,L $

108  MATGPR    GPL,USET,SIL,RUOV//C,N,O $

109  LABEL     LBL9 $

110  SDR1      USET,PG,ULV,UOOV,YS,GO,GM,PS,KFS,KSS,QR/UGV,PGG,QG/V,N,NSKIP/
               C,N,STATICS $

111  CHKPNT    UGV,PGG $

112  COND      LBL8,REPEAT $

113  REPT      LBL11,100 $                          Bottom of DMAP Loop

114  JUMP      ERROR1 $

115  PARAM     //C,N,NOT/V,N,TEST/V,N,REPEAT $

116  COND      ERROR5,TEST $

117  LABEL     LBL8 $

118  CHKPNT    QG $

119  SDR2      CASECC,CSTM,MPT,DIT,EQEXIN,SIL,GPTT,EDT,BGPDT,PGG,QG,UGV,EST,/
               OPG1,OQG1,OUGV1,OES1,OEF1,PUGV1/C,N,STATICS $

120  OFP       OUGV1,OPG1,OQG1,OEF1,OES1,//V,N,CARDNO/V,Y,OPTION $

121  SAVE      CARDNO $

122  COND      P2,JUMPPLOT $

123  PLOT      PLTPAR,GPSETS,ELSETS,CASECC,BGPDT,EQEXIN,SIL,PUGV1, /  PLOTX2/
```

RIGID FORMAT 1

N A S T R A N   S O U R C E   P R O G R A M   C O M P I L A T I O N

```
DMAP-DMAP INSTRUCTION
 NO.
                V,N,NSIL/V,N,LUSET/V,N,JUMPPLOT/V,N,PLTFLG/V,N,PFILE $

124  SAVE      PFILE $

125  PRTMSG    PLOTX2// $

126  LABEL     P2 $

127  JUMP      FINIS $

128  LABEL     ERROR1 $

129  PRTPARM   //C,N,-1/C,N,STATICS $

130  LABEL     ERROR2 $

131  PRTPARM   //C,N,-2/C,N,STATICS $

132  LABEL     ERROR3 $

133  PRTPARM   //C,N,-3/C,N,STATICS $

134  LABEL     ERROR4 $

135  PRTPARM   //C,N,-4/C,N,STATICS $

136  LABEL     ERROR5 $

137  PRTPARM   //C,N,-5/C,N,STATICS $

138  LABEL     FINIS $

139  END       $
```

2.7.2 DMAP Sequence for Nonlinear Steady-State Thermal Analysis

```
RIGID FORMAT DMAP LISTING
SERIES M1

RIGID FORMAT 3 HEAT

   N A S T R A N   S O U R C E   P R O G R A M   C O M P I L A T I O N
DMAP-DMAP INSTRUCTION
 NO.

  1  BEGIN     HEAT NO.03 NON-LINEAR STATIC HEAT TRANSFER ANALYSIS $

  2  GP1       GEOM1,GEOM2,/HGPL,HEQEXIN,HGPDT,HCSTM,HBGPDT,HSIL/V,N,HLUSET/
               C,N,123/V,N,HNOGPDT $

  3  SAVE      HLUSET $

  4  CHKPNT    HGPL,HEQEXIN,HGPDT,HCSTM,HBGPDT,HSIL $

  5  GP2       GEOM2,HEQEXIN/HECT $

  6  CHKPNT    HECT $

  7  PLTSET    PCDB,HEQEXIN,HECT/HPLTSETX,HPLTPAR,HGPSETS,HELSETS/  V,N,HNSIL/
               V,N,JUMPPLOT $

  8  SAVE      HNSIL,JUMPPLOT $

  9  PRTMSG    HPLTSETX//$

 10  SETVAL    //V,N,HPLTFLG/C,N,1/V,N,HPFILE/C,N,0 $

 11  SAVE      HPLTFLG,HPFILE $

 12  COND      HP1,JUMPPLOT $

 13  PLOT      HPLTPAR,HGPSETS,HELSETS,CASECC,HBGPDT,HEQEXIN,HSIL,,/HPLOTX1/
               V,N,HNSIL/V,N,HLUSET/V,N,JUMPPLOT/V,N,HPLTFLG/V,N,HPFILE $

 14  SAVE      JUMPPLOT,HPLTFLG,HPFILE $

 15  PRTMSG    HPLOTX1//$

 16  LABEL     HP1 $

 17  CHKPNT    HPLTPAR,HGPSETS,HELSETS $

 18  GP3       GEOM3,HEQEXIN,GEOM2/HSLT,HGPTT/C,N,123/V,N,HNOGRAV/C,N,123 $

 19  CHKPNT    HSLT,HGPTT $

 20  TA1,      ,HECT,EPT,HBGPDT,HSIL,HGPTT,HCSTM/HEST,,HGEI,HECPT,HGPCT/  V,N,
               HLUSET/C,N,123/V,N,HNOSIMP/C,N,0/V,N,HNOGENL/V,N,HXYZ $

 21  SAVE      HNOSIMP,HNOGENL $

 22  COND      HERROR2,HNOSIMP $

 23  CHKPNT    HEST,HECPT,HGPCT $
```

RIGID FORMAT DMAP LISTING
SERIES M1

RIGID FORMAT 3 HEAT

N A S T R A N   S O U R C E   P R O G R A M   C O M P I L A T I O N

DMAP-NO.	DMAP INSTRUCTION	
24	SMA1	HCSTM,MPT,HECPT,HGPCT,DIT/HKGGX,,HGPST/V,N,HNOGENL/V,N,HNOK4GG/ V,N,HNNLK $
25	SAVE	HNNLK $
26	CHKPNT	HGPST,HKGGX $
27	RMG	HEST,MATPCOL,HGPTT,HKGGX/HRGG,HQGE,HKGG/C,Y,TABS/C,Y,SIGMA=0.0/ V,N,HNLR/V,N,HLLSET $
28	SAVE	HNLR $
29	EQUIV	HKGGX,HKGG/HNLR $
30	PURGE	HQGE,HRGG/HNLR $
31	CHKPNT	HKGG,HQGE,HRGG $
32	GP4	CASECC,GEOM4,HEQEXIN,HSIL,HGPDT/HRG,HYS,HUSET,/V,N,HLLSET/ V,N,HMPCF1/V,N,HMPCF2/V,N,HSINGLE/V,N,HOMIT/V,N,HREACT/ V,N,HNSKIP/V,N,HREPEAT/V,N,HNOSET/V,N,HNOL/V,N,HNOA $
33	SAVE	HMPCF1,HMPCF2,HSINGLE,HOMIT,HREACT,HNSKIP,HREPEAT,HNOSET,HNOL, HNOA $
34	COND	HERROR1,HNOL $
35	PURGE	HGM/HMPCF1/HPS,HKFS,HKSS,HKSF,HRSN,HQG/HSINGLE $
36	EQUIV	HKGG,HKNN/HMPCF1/HRGG,HRNN/HMPCF1 $
37	CHKPNT	HGM,HPS,HKFS,HKSS,HUSET,HRG,HKNN,HRNN,HKSF,HRSN,HYS $
38	GPSP	HGPL,HGPST,HUSET,HSIL/HOGPST $
39	OFP	HOGPST,,,,,//V,N,HCARDNO $
40	SAVE	HCARDNO $
41	COND	HLBL1,HMPCF2 $
42	MCE1	HUSET,HRG/HGM $
43	CHKPNT	HGM $
44	MCE2	HUSET,HGM,HKGG,HRGG,,/HKNN,HRNN,, $
45	CHKPNT	HKNN,HRNN $
46	LABEL	HLBL1 $
47	EQUIV	HKNN,HKFF/HSINGLE/HRNN,HRFN/HSINGLE $

RIGID FORMAT DMAP LISTING
SERIES M1

RIGID FORMAT 3 HEAT

```
   N A S T R A N   S O U R C E   P R O G R A M   C O M P I L A T I O N
DMAP-DMAP INSTRUCTION
 NO.

 48  CHKPNT   HKFF,HRFN $

 49  COND     HLBL2,HSINGLE $

 50  VEC      HUSET/V/C,N,N/C,N,F/C,N,S $

 51  PARTN    HKNN,V,/HKFF,HKSF,HKFS,HKSS $

 52  PARTN    HRNN,,V/HRFN,HRSN,,/C,N,1 $

 53  LABEL    HLBL2 $

 54  CHKPNT   HKFS,HKSS,HKFF,HKSF,HRFN,HRSN $

 55  DECOMP   HKFF/HLLL,HULL/C,N,0/C,N,0/V,N,HMDIAG/V,N,HDET/V,N,HPWR/V,N,
              HKSING $

 56  SAVE     HKSING $

 57  COND     HERROR3,HKSING $

 58  CHKPNT   HLLL,HULL $

 59  SSG1     HSLT,HBGPDT,HCSTM,HSIL,HEST,MPT,HGPTT,EDT,,CASECC,DIT/ HPG/V,N,
              HLUSET/V,N,HNSKIP $

 60  CHKPNT   HPG $

 61  EQUIV    HPG,HPF/HNOSET $

 62  COND     HLBL3,HNOSET $

 63  SSG2     HUSET,HGM,HYS,HKFS,,,HPG/,,HPS,HPF $

 64  LABEL    HLBL3 $

 65  CHKPNT   HPF,HPS $

 66  SSGHT    HUSET,HSIL,HGPTT,HGM,HEST,MPT,DIT,HPF,HPS,HKFF,HKFS,HKSF, HKSS,
              HRFN,HRSN,HLLL,HULL/HUGV,HQG,HRULV/V,N,HNNLK/V,N,HNLR/ C,Y,
              EPSHT=.001/C,Y,TABS=0.0/C,Y,MAXIT=4/V,Y,HIRES=-1/V,N,   HMPCF1/
              V,N,HSINGLE $

 67  CHKPNT   HUGV,HQG,HRULV $

 68  COND     HLBL4,HIRES $

 69  MATGPR   HGPL,HUSET,HSIL,HRULV//C,N,F $

 70  LABEL    HLBL4 $

 71  PLTTRAN  HBGPDT,HSIL/HBGPDP,HSIP/V,N,HLUSET/V,N,HLUSEP $
```

RIGID FORMAT DMAP LISTING
SERIES H1

RIGID FORMAT 3 HEAT

```
   N A S T R A N   S O U R C E   P R O G R A M   C O M P I L A T I O N
DMAP-DMAP INSTRUCTION
 NO.

 72  SAVE      HLUSEP $

 73  CHKPNT    HBGPDP,HSIP $

 74  SDR2      CASECC,HCSTM,MPT,DIT,HEQEXIN,HSIL,HGPTT,EDT,HBGPDP,HPG,HQG,
               HUGV,HEST,/HOPG1,HOQG1,HOUGV1,HOES1,HOEF1,HPUGV1/C,N,STATICS $

 75  OFP       HOUGV1,HOPG1,HOQG1,,,//V,N,HCARDNO $

 76  SAVE      HCARDNO $

 77  SDRHT     HSIL,HUSET,HUGV,HOEF1,HSLT,HEST,DIT,HQGE,,/HOEF1X/C,Y,TABS/
               V,N,HNLR $

 78  OFP       HOEF1X,,,,,//V,N,HCARDNO $

 79  SAVE      HCARDNO $

 80  COND      HP2,JUMPPLOT $

 81  PLOT      HPLTPAR,HGPSETS,HELSETS,CASECC,HBGPDT,HEQEXIN,HSIP,HPUGV1,/
               HPLOTX2/V,N,HNSIL/V,N,HLUSEP/V,N,JUMPPLOT/V,N,HPLTFLG/V,N,
               HPFILE $

 82  PRTMSG    HPLOTX2// $

 83  LABEL     HP2 $

 84  JUMP      FINIS$

 85  LABEL     HERROR1 $

 86  PRTPARM   //C,N,-1/C,N,HSTATICS $

 87  LABEL     HERROR2 $

 88  PRTPARM   //C,N,-2/C,N,HSTATICS $

 89  LABEL     HERROR3 $

 90  PRTPARM   //C,N,-3/C,N,HSTATICS $

 91  LABEL     FINIS$

 92  END       $
```

- *Description of DMAP Operations for Nonlinear Steady-State Thermal Analysis*

2. GP1 generates grid point location tables and tables relating internal and external degree of freedom numbers.

5. GP2 generates the Element Connection Table.

7. PLTSET transforms the input data into plot data tables.

9. PRTMSG prints error messages associated with the plot data.

13. PLØT generates all plots of the structure without temperature profiles.

15. PRTMSG prints plotter and engineering data for each generated plot.

18. GP3 generates applied heat flux load tables (SLT) and the grid point temperature table.

20. TA1 generates element tables for use in matrix formulation, load generation, and element heat flux data recovery.

24. SMA1 generates the conductivity matrix, $[K^x_{gg}]$, and the grid point singularity table.

27. RMG generates the radiation matrix, $[R_{gg}]$, and adds the estimated linear component of radiation to the conductivity matrix. The element radiation flux matrix, $[Q_{ge}]$, is also generated for use in recovery data for the HBDY elements.

32. GP4 generates flags defining member of various displacement sets (USET) and forms multi-point constraint equations $[R_g]\ \{u_g\} = \{0\}$.

34. Go to DMAP instruction 85 if no independent degrees of freedom are defined.

38. GPSP determines if possible matrix singularities remain. These may be extraneous in a radiation problem, since some points may transfer heat through radiation only.

39. ØFP prints the singularity messages.

41. Go to DMAP statement 46 if no multi-point constraints exist.

42. MCE1 partitions the multi-point constraint equation matrix $[R_g] = [R_m \mid R_n]$ and solves for the multi-point constraint transformation matrix

$$[G_m] = -\,[R_m]^{-1}\,[R_n].$$

44. MCE2 partitions conductivity and radiation matrices

$$[K_{gg}] = \left[\begin{array}{c|c} \bar{K}_{nn} & K_{nm} \\ \hline K_{mn} & K_{mm} \end{array}\right] \quad \text{and} \quad [R_{gg}] = \left[\begin{array}{c|c} \bar{R}_{nn} & R_{nm} \\ \hline R_{mn} & R_{mm} \end{array}\right]$$

and performs matrix reductions

$$[K_{nn}] = [\bar{K}_{nn}] + [G_m^T]\,[K_{mn}] + [K_{mn}^T]\,[G_m] + [G_m^T]\,[K_{mm}]\,[G_m] \text{ and}$$

$$[R_{nn}] = [\bar{R}_{nn}] + [G_m^T]\,[R_{mn}] + [R_{mn}^T]\,[G_m] + [G_m^T]\,[K_{mm}]\,[G_m]$$

47. Equivalence $[K_{ff}]$ to $[K_{nn}]$ and $[R_{fn}]$ to $[R_{nn}]$ if no single-point constraints exist.

49. Go to DMAP statement 53 if no single-point constraints exist.

50. VEC is used to generate a partitioning vector $u_n \rightarrow u_f + u_s$.

51. PARTN is used to partition the conductivity matrix

$$[K_{nn}] = \left[\begin{array}{c|c} K_{ff} & K_{fs} \\ \hline K_{fs} & K_{ss} \end{array}\right]$$

52. PARTN is used to partition the radiation matrix

$$[R_{nn}] = \left[\begin{array}{c} R_{fn} \\ \hdashline R_{sn} \end{array}\right]$$

55. DECØMP decomposes the potentially unsymmetric matrix K_{ff} into upper and lower triangular factors $[U_{\ell\ell}]$ and $[L_{\ell\ell}]$.

57. Go to DMAP statement 89 if the matrix is singular.

59. SSG1 is used to generate the input heat flux vector $\{P_g\}$.

62. Go to DMAP statement 64 if no constraints of any kind exist.

63. SSG2 reduces the heat flux vector

$$\{P_g\} = \left\{\begin{array}{c} \bar{P}_n \\ \hdashline P_m \end{array}\right\}$$

$$\{P_n\} = \{\bar{P}_n\} + [G_m^T]\{P_m\}$$

$$\{P_n\} = \left\{\begin{array}{c} P_f \\ \hdashline P_s \end{array}\right\}$$

66. SSGHT solves the nonlinear heat transfer problems by iteration. It uses user input parameters EPSHT and MAXIT to limit the iterations. For details, refer to Section 8 of the NASTRAN Theoretical Manual. The output data blocks are: $\{u_g\}$, the solution temperature vector, $\{q_g\}$, the heat flux due to single point constraints, and $\{\delta P_\ell\}$, the matrix of residual heat fluxes at each iteration step.

68. Go to DMAP statement 70 if no residual vectors are desired.

69. MATGPR prints the matrix of residual vectors.

71. PLTTRAN transforms the grid point definition tables into a format for plotting temperature solutions.

74. SDR2 calculates the heat flux due to conductivity and convection in the elements and prepares the solution vectors for output.

75. ØFP formats tables prepared by SDR2 for output.

77. SDRHT processes the HBDY elements to produce heat flux into the elements due to convection, radiation, and user applied flux.

78. ØFP formats the output element flux table for output.

80. Go to DMAP 83 if no temperature profile plots are requested.

81. PLØT generates temperature profile plots.

82. PRTMSG prints plotter data and engineering data for each plot generated.

84. Go to DMAP 92 and make normal exit.

86. NØNLINEAR STATIC HEAT TRANSFER ANALYSIS ERRØR MESSAGE NØ. 1 - NØ INDEPENDENT DEGREES ØF FREEDØM HAVE BEEN DEFINED.

88. NØNLINEAR STATIC HEAT TRANSFER ANALYSIS ERRØR MESSAGE NØ. 2 - NØ SIMPLE STRUCTURAL ELEMENTS.

90. NØNLINEAR STATIC HEAT TRANSFER ANALYSIS ERRØR MESSAGE NØ. 3 - STIFFNESS MATRIX SINGULAR.

2.7.3 DMAP Sequence for Transient Thermal Analysis

```
RIGID FORMAT DMAP LISTING
SERIES M1

RIGID FORMAT 9 HEAT

   N A S T R A N   S O U R C E   P R O G R A M   C O M P I L A T I O N
DMAP-DMAP INSTRUCTION
NO.

 1  BEGIN     HEAT NO.9 TRANSIENT HEAT TRANSFER ANALYSIS $

 2  FILE      KGGX=TAPE/ KGG=TAPE $

 3  GP1       GEOM1,GEOM2,/HGPL,HEQEXIN,HGPDT,HCSTM,HBGPDT,HSIL/V,N,HLUSET/
              V,N,HALWAYS=-1/V,N,HNOGPDT $

 4  SAVE      HLUSET,HNOGPDT$

 5  PURGE     HUSET,HGM,HGC,HKAA,HBAA,HPSO,HKFS,HQP,HEST/HNOGPDT $

 6  CHKPNT    HGPL,HEQEXIN,HGPDT,HCSTM,HBGPDT,HSIL,HUSET,HGM,HGC,HKAA,HBAA,
              HPSO,HKFS,HQP,HEST $

 7  COND      HLBL5,HNOGPDT$

 8  GP2       GEOM2,HEQEXIN/HECT $

 9  CHKPNT    HECT $

10  PLTSET    PCDB,HEQEXIN,HECT/HPLTSETX,HPLTPAR,HGPSETS,HELSETS/V,N,HNSIL/V,
              N,JUMPPLOT $

11  SAVE      HNSIL,JUMPPLOT $

12  PRTMSG    HPLTSETX//$

13  SETVAL    //V,N,HPLTFLG/C,N,1/V,N,HPFILE/C,N,0 $

14  SAVE      HPLTFLG,HPFILE $

15  COND      HP1,JUMPPLOT$

16  PLOT      HPLTPAR,HGPSETS,HELSETS,CASECC,HBGPDT,HEQEXIN,HSIL,,/HPLOTX1/
              V,N,HNSIL/V,N,HLUSET/V,N,JUMPPLOT/V,N,HPLTFLG/V,N,HPFILE $

17  SAVE      JUMPPLOT,HPLTFLG,HPFILE $

18  PRTMSG    HPLOTX1//$

19  LABEL     HP1 $

20  CHKPNT    HPLTPAR,HGPSETS,HELSETS $

21  GP3       GEOM3,HEQEXIN,GEOM2/HSLT,HGPTT/C,N,123/C,N,123/C,N,123 $

22  CHKPNT    HGPTT,HSLT $

23  TA1,      ,HECT,EPT,HBGPDT,HSIL,HGPTT,HCSTM/HEST,,HGEI,HECPT,HGPCT/  V,N,
              HLUSET/C,N,123/V,N,HNOSIMP=-1/C,N,0/C,N,123/C,N,123 $
```

RIGID FORMAT DMAP LISTING
SERIES M1

RIGID FORMAT 9 HEAT

```
   N A S T R A N   S O U R C E   P R O G R A M   C O M P I L A T I O N
DMAP-DMAP INSTRUCTION
 NO.

 24  SAVE     HNOSIMP $

 25  CHKPNT   HEST,HECPT,HGPCT $

 26  COND     HLBL1,HNOSIMP$

 27  SMA1     HCSTM,MPT,HECPT,HGPCT,DIT/HKGGX,,HGPST/C,N,123/C,N,123/V,N,
              HNNLK $

 28  SAVE     HNNLK $

 29  CHKPNT   HKGGX,HGPST $

 30  SMA2     HCSTM,MPT,HECPT,HGPCT,DIT/,HBGG/C,N,1.0/C,N,123/V,N,   HNOBGG=
              -1/C,N,-1 $

 31  SAVE     HNOBGG $

 32  PURGE    HBNN,HBFF,HBAA,HBGG/HNOBGG$

 33  CHKPNT   HBGG,HBNN,HBFF,HBAA $

 34  LABEL    HLBL1 $

 35  RMG      HEST,MATPOOL,HGPTT,HKGGX/HRGG,HQGE,HKGG/C,Y,TABS/C,Y,SIGMA=0.0/
              V,N,HNLR/V,N,HLUSET $

 36  SAVE     HNLR $

 37  EQUIV    HKGGX,HKGG/HNLR $

 38  PURGE    HRGG,HRNN,HRFF,HRAA,HRDD/HNLR $

 39  CHKPNT   HRGG,HRNN,HRFF,HRAA,HRDD,HKGG,HQGE $

 40  GP4      CASECC,GEOM4,HEQEXIN,HSIL,HGPDT/HRG,,HUSET,/V,N,HLUSET/V,N,
              HMPCF1=-1/V,N,HMPCF2=-1/V,N,HSINGLE=-1/V,N,HOMIT=-1/V,N,HREACT=
              -1/C,N,0/C,N,123/V,N,HNOSET=-1/V,N,HNOL/V,N,HNOA=-1 $

 41  SAVE     HMPCF1,HSINGLE,HOMIT,HNOSET,HREACT,HMPCF2,HNOL,HNOA $

 42  PURGE    HGM,HGMD/HMPCF1/HGO,HGOD/HOMIT/HKFS,HPSO,HQP/HSINGLE $

 43  EQUIV    HKGG,HKNN/HMPCF1/HRGG,HRNN/HMPCF1/HBGG,HBNN/HMPCF1 $

 44  CHKPNT   HGM,HRG,HGO,HKFS,HQP,HUSET,HGMD,HGOD,HPSO,HKNN,HRNN,HBNN $

 45  COND     HLBL2,HNOSIMP $

 46  GPSP     HGPL,HGPST,HUSET,HSIL/HOGPST $

 47  OFP      HOGPST,,,,,//V,N,HCARDNO $
```

RIGID FORMAT DMAP LISTING
SERIES M1

RIGID FORMAT 9 HEAT

N A S T R A N   S O U R C E   P R O G R A M   C O M P I L A T I O N

```
DMAP-DMAP INSTRUCTION
NO.

48  SAVE     HCARDNO $

49  LABEL    HLBL2 $

50  COND     HLBL3,HMPCF1 $

51  MCE1     HUSET,HRG/HGM $

52  CHKPNT   HGM $

53  MCE2     HUSET,HGM,HKGG,HRGG,HBGG,/HKNN,HRNN,HBNN, $

54  CHKPNT   HKNN,HRNN,HBNN $

55  LABEL    HLBL3 $

56  EQUIV    HKNN,HKFF/HSINGLE/HRNN,HRFF/HSINGLE/HBNN,HBFF/HSINGLE $

57  CHKPNT   HKFF,HRFF,HBFF $

58  COND     HLBL4,HSINGLE $

59  SCE1     HUSET,HKNN,HRNN,HBNN,/HKFF,HKFS,,HRFF,HBFF, $

60  CHKPNT   HKFS,HKFF,HRFF,HBFF $

61  LABEL    HLBL4 $

62  EQUIV    HKFF,HKAA/HOMIT/HRFF,HRAA/HOMIT/HBFF,HBAA/HOMIT $

63  CHKPNT   HKAA,HRAA,HBAA $

64  COND     HLBL5,HOMIT $

65  SMP1     HUSET,HKFF,,,/HGO,HKAA,,,,,,,, $

66  CHKPNT   HGO,HKAA $

67  COND     HLBLR,HNLR $

68  SMP2     HUSET,HGO,HRFF/HRAA $

69  CHKPNT   HRAA $

70  LABEL    HLBLR $

71  COND     HLBL5,HNOBGG $

72  SMP2     HUSET,HGO,HBFF/HBAA $

73  CHKPNT   HBAA $
```

RIGID FORMAT DMAP LISTING
SERIES M1

RIGID FORMAT 9 HEAT

N A S T R A N   S O U R C E   P R O G R A M   C O M P I L A T I O N

```
DMAP-DMAP INSTRUCTION
 NO.

 74  LABEL    HLBL5 $

 75  DPD      DYNAMICS,HGPL,HSIL,HUSET/HGPLD,HSILD,HUSETD,HTFPOOL,HDLT,,,
              HNLFT,HTRL,,HEQDYN/V,N,HLUSET/V,N,HLUSETD/C,N,123 /V,N,HNODLT/
              C,N,123/C,N,123/V,N,HNONLFT/V,N,HNOTRL/C,N,123/C,N,123/ V,N,
              HNOUE $

 76  SAVE     HLUSETD,HNODLT,HNONLFT,HNOTRL,HNOUE $

 77  COND     HERROR1,HNOTRL$

 78  EQUIV    HGO,HGOD/HNOUE/HGM,HGMD/HNOUE $

 79  PURGE    HPPO,HPSO,HPDO,HPDT/HNODLT $

 80  CHKPNT   HUSETD,HEQDYN,HTFPOOL,HDLT,HTRL,HGOD,HGMD,HNLFT,HSILD,HGPLD,
              HPPO,HPSO,HPDO,HPDT $

 81  MTRXIN   CASECC,MATPOOL,HEQDYN,,HTFPOOL/HK2PP,,HB2PP/V,N,HLUSETD/ V,N,
              HNOK2PP/C,N,123/V,N,HNOB2PP $

 82  SAVE     HNOK2PP,HNOB2PP $

 83  PARAM    //C,N,AND/V,N,HKDEKA/V,N,HNOUE/V,N,HNOK2PP $

 84  PURGE    HK2DD/HNOK2PP/HB2DD/HNOB2PP $

 85  EQUIV    HKAA,HKDD/HKDEKA/HB2PP,HB2DD/HNOA/HK2PP,HK2DD/HNOA/HRAA,HRDD/
              HNOUE $

 86  CHKPNT   HK2PP,HB2PP,HK2DD,HB2DD,HKDD,HRDD $

 87  COND     HLBL6,HNOGPDT $

 88  GKAD     HUSETD,HGM,HGO,HKAA,HBAA,HRAA,,HK2PP,,HB2PP/HKDD,HBDD, HRDD,
              HGMD,HGOD,HK2DD,HM2DD,HB2DD/C,N,TRANRESP/C,N,DISP/C,N, DIRECT/
              C,Y,HG=0.0/C,Y,HW3=0.0/C,Y,HW4=0.0/V,N,HNOK2PP/C,N,-1/   V,N,
              HNOB2PP/V,N,HMPCF1/V,N,HSINGLE/V,N,HOMIT/V,N,HNOUE/ C,N,-1/V,N,
              HNOBGG/V,N,HNOSIMP/C,N,-1 $

 89  LABEL    HLBL6 $

 90  EQUIV    HK2DD,HKDD/HNOSIMP/HB2DD,HBDD/HNOGPDT $

 91  CHKPNT   HKDD,HBDD,HRDD,HGMD,HGOD $

 92  TRLG     CASECC,HUSETD,HDLT,HSLT,HBGPDT,HSIL,HCSTM,HTRL,DIT,HGMD,HGOD,,
              HEST/HPPO,HPSO,HPDO,HPDT,,HTOL/V,N,HNOSET/V,N,HPDEPDO $

 93  SAVE     HPDEPDO,HNOSET $

 94  EQUIV    HPPO,HPDO/HNOSET $
```

RIGID FORMAT DMAP LISTING
SERIES M1

RIGID FORMAT 9 HEAT

```
   N A S T R A N   S O U R C E   P R O G R A M   C O M P I L A T I O N
DMAP-DMAP INSTRUCTION
 NO.

 95  EQUIV     HPDO,HPDT/HPCEPDC $

 96  CHKPNT    HPPO,HPDO,HPSO,HTCL,HPDT $

 97  TRHT      CASECC,HUSETD,HNLFT,DIT,HGPTT,HKDD,HBDD,HRDD,HPDT,HTRL/HUDVT,
               HPNLD/C,Y,BETA=.55/C,Y,TABS=0.0/V,N,HNLR/C,Y,RADLIN=-1 $

 98  CHKPNT    HUDVT,HPNLD $

 99  VDR       CASECC,HECDYN,HUSETD,HUDVT,HTOL,XYCDB,HPNLD/HOUDV1,HOPNL1/  C,
               N,TRANRESP/C,N,DIRECT/C,N,0/V,N,HNOD/V,N,HNOP/C,N,C $

100  SAVE      HNOD,HNOP $

101  CHKPNT    HOUDV1,HOPNL1 $

102  COND      HLBL7,HNOD $

103  SDR3      HOUDV1,HOPNL1,,,,/HOUDV2,HOPNL2,,,, $

104  OFP       HOUDV2,HOPNL2,,,,//V,N,HCARDNO $

105  SAVE      HCARDNO $

106  CHKPNT    HOPNL2,HOUDV2 $

110  LABEL     HLBL7 $

111  PARAM     //C,N,AND/V,N,HPJUMP/V,N,HNOP/V,N,JUMPPLOT $

112  COND      HLBL9,HPJUMP $

113  EQUIV     HUDVT,HUPV/HNOA $

114  COND      HLBL8,HNOA $

115  SDR1      HUSETD,,HUDVT,,,HGOD,HGMD,HPSO,HKFS,,/HUPV,,HQP/C,N,1/C,N,
               TRANSNT $

116  LABEL     HLBL8 $

117  CHKPNT    HUPV,HQP $

118  PLTTRAN   HBGPDT,HSIL/HBGPDP,HSIP/V,N,HLUSET/V,N,HLLSEP $

119  SAVE      HLUSEP $

120  SDR2      CASECC,HCSTM,MPT,DIT,HEQDYN,HSILD,,HTOL,HBGPDP,HPPC,HCP,HUPV,
               HEST,XYCDB/HOPP1,HOQP1,HOUPV1,HOES1,HOEF1,HPUGV /C,N,
               TRANRESP $

121  SDR3      HOPP1,HOQP1,HOUPV1,HOES1,HOEF1,/HOPP2,HOQP2,HOUPV2,HOES2,
```

RIGID FORMAT DMAP LISTING
SERIES M1

RIGID FORMAT 9 HEAT

N A S T R A N   S O U R C E   P R O G R A M   C O M P I L A T I O N

```
DMAP-DMAP INSTRUCTICN
 NO.
                HOEF2, $

122  CHKPNT     HUPP2,HOCF2,HOUPV2,HOES2,HOEF2 $

123  OFP        HOPP2,HOCP2,HOUPV2,HOEF2,HOES2,//V,N,HCARDNO $

124  SAVE       HCARDNO $

125  COND       HP2,JUMPPLOT $

126  PLOT       HPLTPAR,HGPSETS,HELSETS,CASECC,HBGPDT,HEQEXIN,HSIP,,HPUGV/
                HPLOTX2/V,N,HNSIL/V,N,HLUSEP/V,N,JUMPPLOT/V,N,HPLTFLG/V,N,
                HPFILE $

127  SAVE       HPFILE $

128  PRTMSG     HPLOTX2// $

129  LABEL      HP2 $

130  XYTRAN     XYCDB,HOPP2,HOCP2,HOUPV2,HOES2,HOEF2/HXYPLTT/C,N,TRAN/C,N,PSET/
                V,N,HPFILE/V,N,HCARDNO $

131  SAVE       HPFILE,HCARDNO $

132  XYPLOT     HXYPLTT// $

133  LABEL      HLBL9 $

134  JUMP       FINIS $

135  LABEL      HERROR1 $

136  PRTPARM    //C,N,-1/C,N,HDIRTRD$

137  LABEL      FINIS$

138  END        $
```

- *Description of DMAP Operations for Transient Thermal Analysis*

3. GP1 generates grid point location tables and tables relating internal and external degree of freedom indices.

7. Go to DMAP 74 if only direct matrix input.

8. GP2 generates the Element Connection Table.

10. PLTSET transforms user input into plot data tables.

12. PRTMSG prints error messages associated with the structure plotter.

15. Go to DMAP 19 if no structure-only plots are requested.

16. PLØT generates all plots of the structure without temperature profiles.

18. PRTMSG prints plotter data and engineering data for each generated plot.

21. GP3 generates the table of user defined temperature sets and the tables of static heat flux input data.

23. TA1 generates element tables for use in matrix formulation, load generation, and element data recovery.

26. Go to DMAP 34 if no structural or boundary elements exist.

27. SMA1 generates the conductivity matrix, $[K^x_{gg}]$, and the grid point singularity table.

30. SMA2 generates the heat capacity matrix, $[B_{gg}]$.

35. RMG generates the radiation matrix, $[R_{gg}]$, and adds the estimated linear component of radiation to the conductivity matrix. The element-radiation flux matrix, $[Q_{ge}]$, is also generated for use in data recovery.

37. Equivalence the linear heat transfer matrix, $[K_{gg}]$, to the conductivity matrix if no radiation exists.

40. GP4 generates flags defining members of various displacement sets (USET) and forms the multi-point constraint equations, $[R_g]\{u_g\} = 0$.

43. Equivalence $[K_{nn}]$ to $[K_{gg}]$, $[R_{nn}]$ to $[R_{gg}]$, and $[B_{nn}]$ to $[B_{gg}]$ if no multi-point constraints exist.

45. Go to DMAP 49 if no simple elements exist.

46. GPSP determines if possible matrix singularities remain. These may be extraneous in a radiation problem, since some points may transfer heat through radiation only.

47. ØFP prints the singularity messages.

50. Go to DMAP 55 if no multi-point constraints exist.

51. MCE1 partitions the multi-point constraint equation matrix, $[R_g] = [R_m \vdots R_n]$, and solves for the multi-point constraint transformation matrix,

$$[G_m] = -[R_m]^{-1}[R_n] .$$

53. MCE2 partitions conductivity and radiation matrices

$$[K_{gg}] = \left[\begin{array}{c|c} \bar{K}_{nn} & K_{nm} \\ \hline K_{mn} & K_{mm} \end{array}\right]$$

$$[R_{gg}] = \left[\begin{array}{c|c} \bar{R}_{nn} & R_{nm} \\ \hline R_{mn} & R_{mm} \end{array}\right]$$

$$B_{gg} = \left[\begin{array}{c|c} \bar{B}_{nn} & B_{nm} \\ \hline B_{mn} & B_{mm} \end{array}\right]$$

and performs matrix reductions

$$[K_{nn}] = [\bar{K}_{nn}] + [G_m^T]\,[K_{mn}] + [K_{mn}]\,[G_m] + [G_m^T]\,[K_{mm}]\,[G_m].$$

The same equation is applied to R_{nn} and B_{nn}.

56. Equivalence $[K_{ff}]$ to $[K_{nn}]$, $[B_{ff}]$ to $[B_{nn}]$, and $[R_{ff}]$ to $[R_{nn}]$ if no single point constraints exist.

58. Go to DMAP 61 if no single point constraints exist.

59. SCE1 partitions the matrices as follows:

$$[K_{nn}] = \left[\begin{array}{c|c} K_{ff} & K_{fs} \\ \hline K_{sf} & K_{ss} \end{array}\right]$$

R_{nn} and B_{nn} are partitioned in the same manner, except only the ff partitions are saved.

62. Equivalence $[K_{aa}]$ to $[K_{ff}]$, $[R_{aa}]$ to $[R_{ff}]$, and $[B_{aa}]$ to $[B_{ff}]$ if no omitted coordinates are requested.

64. Go to DMAP 74 if no omitted coordinates are requested.

65. SMP1 partitions the conductivity matrix

$$[K_{ff}] = \left[\begin{array}{c|c} \bar{K}_{aa} & K_{ao} \\ \hline K_{oa} & K_{oo} \end{array}\right]$$

solves for the transformation matrix $[G_o]$:

$$[K_{oo}]\,[G_o] = -\,[K_{oa}]$$

and solves for the reduced conductivity matrix $[K_{aa}]$:

$$[K_{aa}] = [\bar{K}_{aa}] + [K_{ao}]\,[G_o]$$

67. Go to DMAP 70 if no radiation matrix exists.

68. SMP2 partitions constrained radiation matrix

$$[R_{ff}] = \left[\begin{array}{c|c} \bar{R}_{aa} & R_{ao} \\ \hline R_{oa} & R_{oo} \end{array}\right]$$

and performs matrix reduction

$$[R_{aa}] = [\bar{R}_{aa}] + [R_{oa}^T]\,[G_o] + [G_o^T]\,[R_{oa}] + [G_o^T]\,[R_{oo}]\,[G_o]$$

71. Go to DMAP 74 if no heat capacity matrix, $[B_{ff}]$, exists.

72. SMP2 calculates a reduced heat capacity matrix, $[B_{aa}]$, with the same equation as Step 68.

75. DPD generates the table defining the displacement sets each degree of freedom belongs to (USETD), including extra points. It prepares the Transfer Function Pool, the Dynamics Load Table, the Nonlinear Function Table, and the Transient Response List.

77. Go to DMAP 135 and exit if no time step data was specified.

78. Equivalence $[G_o^d]$ to $[G_o]$ and $[G_m^d]$ to $[G_m]$ if no extra points were defined.

81. MTRXIN selects the direct input matrices $[K_{pp}^2]$ and $[B_{pp}^2]$.

85. Equivalence $[K_{dd}^1]$ to $[K_{aa}]$ if no direct input stiffness matrices and no extra points; $[B_{dd}^2]$ to $[B_{pp}]$ and $[K_{dd}^2]$ to $[K_{pp}]$ if only extra points are used; and $[R_{dd}]$ to $[R_{aa}]$ if no extra points are used.

87. Go to DMAP 89 if no structure was defined.

88. GKAD expands the matrices to include extra points and assembles conductivity, capacitance, and radiation matrices for use in Direct Transient Response.

$$[K_{dd}^1] = \left[\begin{array}{c|c} K_{aa} & 0 \\ \hline 0 & 0 \end{array}\right]$$

$$[B_{dd}^1] = \left[\begin{array}{c|c} B_{aa} & 0 \\ \hline 0 & 0 \end{array}\right]$$

$$[R_{dd}] = \left[\begin{array}{c|c} R_{aa} & 0 \\ \hline 0 & 0 \end{array}\right]$$

$$[K_{dd}] = [K^1_{dd}] + [K^2_{dd}]$$

$$[B_{dd}] = [B^1_{dd}] + [B^2_{dd}]$$

(Nonzero values of the parameters W4, G, and W3 are not recommended for use in heat transfer analysis.)

90. Equivalence $[K_{dd}]$ to $[K^2_{dd}]$ and $[B_{dd}]$ to $[B^2_{dd}]$ if no matrices were generated from the structural elements.

92. TRLG generates matrices of heat flux loads versus time. $\{P^o_p\}$, $\{P^o_s\}$, and $\{P^o_d\}$ are generated with one column per output time step. $\{P^t_d\}$ is generated with one column per solution time step, and the Transient Output List is a list of output time steps.

94. Equivalence $\{P^o_d\}$ to $\{P^o_p\}$ if the d and p sets are the same.

95. Equivalence $\{P^t_d\}$ to $\{P^o_d\}$ if the output times are the same as the solution times.

97. TRHT integrates the equation of motion:

$$[B_{dd}]\,\{\dot{u}\} + [K_{dd}]\,\{u\} = \{P_d\} + \{N_d\}$$

where $\{u\}$ is a vector of temperatures at any time,
$\{\dot{u}\}$ is the time derivative of $\{u\}$ ("velocity"),
$\{P_d\}$ is the applied heat flux at any time step, and
$\{N_d\}$ is the total nonlinear heat flux from radiation and/or NØLIN data, extrapolated from the previous solution vector.

The output consists of the $[u^t_d]$ matrix containing temperature vectors and temperature "velocity" vectors for the output time steps.

99. VDR processes the user solution set output requests.

102. Go to DMAP 110 if no solution set output is desired.

103. SDR3 transforms the requested temperature and nonlinear load values into output SØRT2 format.

104. ØFP formats the temperature, temperature velocity, and heat flux nonlinear loads for printout.

112. Go to DMAP 133 and exit if no further output is desired.

113. Equivalence $[u_d]$ to $[u_p]$ if no structure points were input.

114. Go to DMAP 116 if no structure points were input.

115. SDR1 recovers the dependent temperatures:

$$\{u_o\} = [G_o^d]\ \{u_d\}$$

$$\left\{\begin{array}{c} u_d \\ \hline u_o \end{array}\right\} = \{u_f\}$$

$$\left\{\begin{array}{c} u_f+u_e \\ \hline u_s=0 \end{array}\right\} = \{u_n\}$$

$$\{u_m\} = [G_m^d]\ \{u_f+u_e\}$$

$$\left\{\begin{array}{c} u_n+u_e \\ \hline u_m \end{array}\right\} = \{u_p\}$$

The module also recovers the heat flux into the points having single-point constraints.

$$\{q_s\} = -\ \{P_s\} + [K_{fs}^T]\ \{u_f\}$$

118. PLTTRAN coverts the grid point tables to standard plot form when grid points with one degree of freedom are used.

120. SDR2 calculates requested heat flux transfer in the elements and transforms temperatures, velocities, and heat flux loads into output form.

121. SDR3 prepares requested output in SØRT2 order.

123. ØFP formats requested output and places it on the system output file.

126. PLØT generates plots of the temperature profile on the structure for specified times.

128. PRTMSG prints plotter data and engineering data for structure plots.

130. XYTRAN prepares tables of requested grid point or element output quantities for XYPLØT.

132. XYPLØT prepares requested plots of temperatures, velocities, element flux, or applied heat loads versus time.

136. TRANSIENT HEAT TRANSFER ANALYSIS ERRØR MESSAGE NØ. 1 - TRANSIENT RESPØNSE LIST REQUIRED FØR TRANSIENT RESPØNSE CALCULATIØNS.

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3. THE NASTRAN THERMAL ANALYZER INPUT DATA DECK

3.1 Overview of Thermal Model Preparation

The NASTRAN Thermal Analyzer is a general heat transfer computer program using a discrete finite element approach, wherein heat conducting solids are represented by a model consisting of a finite number of idealized heat conduction elements that are interconnected at a finite number of grid points. Various forms of heat conduction elements are provided to represent common structural members including 1-D rods, 2-D triangular and quadrilateral plates and axisymmetric rings of triangular and trapezoidal cross-sections, and 3-D solids such as wedges, tetrahedra and hexahedra. Also included are scalar heat conduction elements which are flexible and serve as lumped thermal conductors connecting pairs of grid points with appropriate conductances.

Different types of thermal loads, in both static and dynamic modes, may be applied directly to grid points, or indirectly via the heat boundary element. The primary types of thermal loads included are concentrated loads applied to the points, internally generated heat within an element volume, and uniform heat fluxes as well as directional heat sources applied to the surface of an element.

A constant or temperature-dependent (in NLSS only) convective film coefficient may be used to specify the convective coupling between a surface and an ambient temperature. The capability to analyze radiative energy exchanges among surfaces within a generalized enclosure is also provided. To facilitate the solution of different types of problems. Three rigid formats are provided: (1) Linear steady-state (LSS) analysis, (2) Nonlinear steady-state (NLSS) analysis, and (3) Transient thermal analysis. The functional flow of bulk data cards used by this heat transfer computer program relative to the definition, constraints and thermal loadings of the finite element thermal model is shown in figure 3.1. This Bulk Data Deck constitutes the main portion of a complete $\overline{\text{N}}$TA input deck.

The grid point definition forms the basic framework for the discretized thermal model. All other parts of the model structure are referenced either directly or indirectly to the grid points.

Two general types of grid points are used in defining the finite element thermal model. They are:

1. Geometric grid point – a point in three-dimensional space at which a temperature is defined (as six degrees of freedom were provided originally in the structural version of NASTRAN to facilitate three components of translation and three components of rotation, only the first component is selected for temperature representation). The coordinates of each grid point are specified by the user, but may be omitted if the location is irrelevant.

2. Scalar point – a point in vector space, at which a temperature is defined, can be coupled to geometric grid points by means of scalar elements or by constraint relationships. However, the use of this type of point is not recommended.

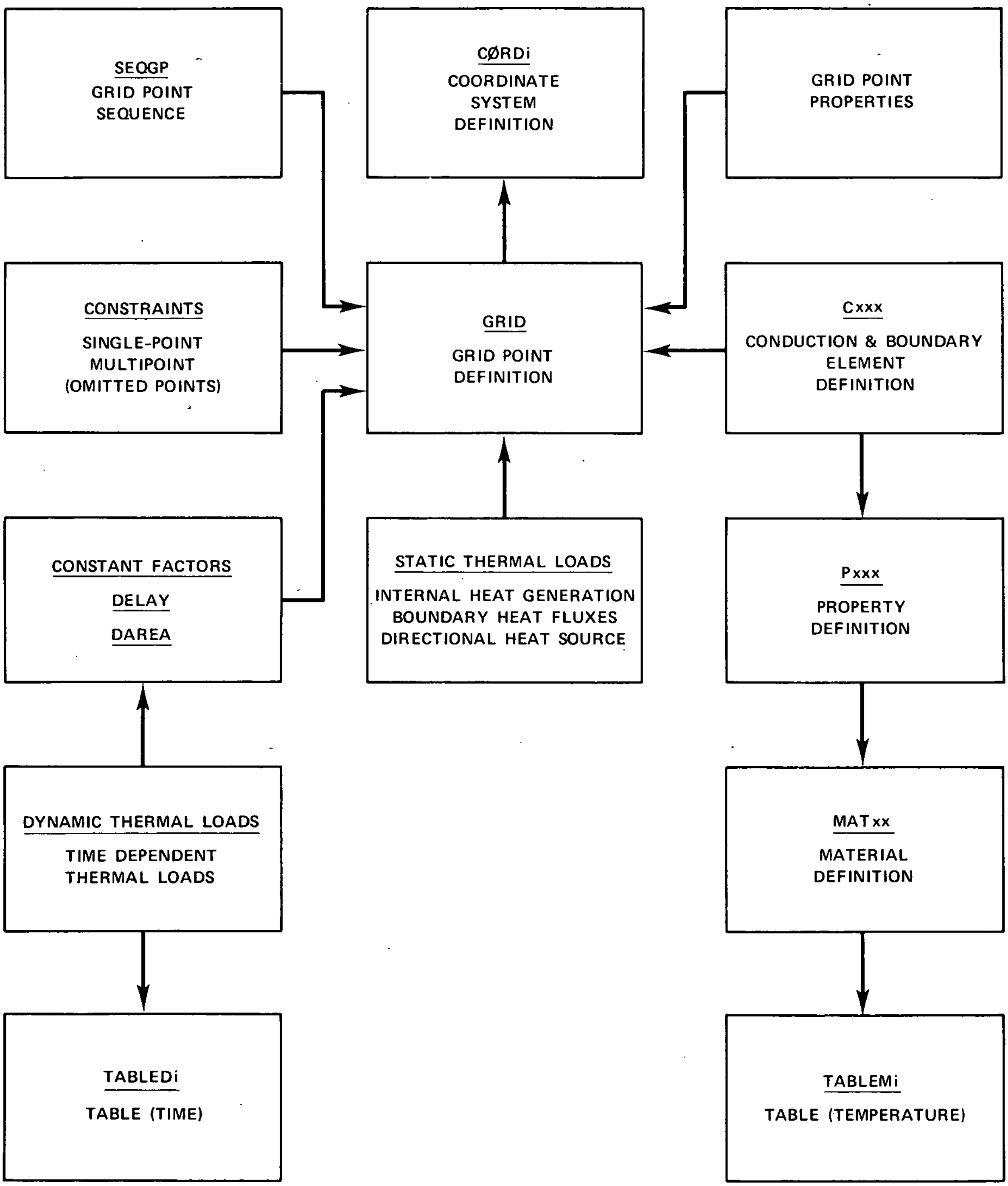


Figure 3.1. Thermal model functional diagram.

Heat conduction elements are defined on connection cards by referencing grid points. In a few cases, all of the information required to generate the conductivity matrix for the element is given on the connection card. In most cases the connection card refers to a property card, on which the cross-sectional properties of the element are given. The property card in turn refers to a material card which gives the material properties. If some of the material properties are temperature dependent, a further reference is made to tables for this information.

The heat boundary element, as defined on the connection card (CHBDY) by referencing grid points, is provided to accept external heat fluxes input to a bounding surface. It is also used to model boundary surfaces of the heat conduction elements when thermal convection and radiative exchanges are present. The associated property card (PHBDY) is often required to provide data specifying surface areas and emissivities. The radiative surface properties, emissivity and absorptivity, are given on the property card. This card is also used together with the thermal material card (MAT4) if the convective film coefficient and the thermal capacity of the boundary film are being defined.

The heat boundary element provides six optional surface configurations with the following characteristics:

1. The "POINT" type has one primary grid point forming the CHBDY card and requires a PHBDY card to specify the value of the POINT area. The surface normal of the represented POINT surface must be given if directional thermal flux is being used as an input.

2. The "LINE" type has two primary grid points forming the CHBDY card and requires a PHBDY card to specify the width of the LINE. The surface normal of the represented surface must be given if an input is a directional thermal flux.

3. The "REV" type has two primary grid points forming the CHBDY card. The defined area is a conical section formed by rotating the line defined by the two grid points around the Z-axis of the basic coordinate system.

4. The "AREA3" and "AREA4" types have three and four primary grid points forming the CHBDY cards, respectively. These grid points define a triangular or quadrilateral surface and must be ordered in one direction around the boundary. An outward surface normal is implied following the sequencing order of the grid points according to the right-hand screw rule. A PHBDY card is required for convective and/or radiative exchanges and/or directional thermal flux.

5. The "ELCYL" type (elliptic cylinder) has two primary grid points, and requires a PHBDY card to specify two radii of the elliptic cylinder and an effective width.

Various kinds of constraints can be applied to the grid points. Single-point constraints (SPC) are used in steady-state thermal analyses to specify prescribed temperatures at grid points. Multipoint constraints (MPC) are used to specify a linear relationship among selected grid point temperatures, including the simulation of perfect conductor elements. Omitted points,

when available, are to be used in matrix partitioning and to reduce the number of unknown temperatures in transient thermal analysis.

Static thermal loads consisting of the internally generated heat, boundary heat fluxes and heat input from directional heat sources are provided to represent various types of heat inputs. The heat boundary element proportionates the total energy received by the boundary element surface to the vertices of that element according to its shape and size. The same loads may be modified by a time function to become time-dependent dynamic thermal loads.

In regard to compatibility with structural analysis, the finite element thermal model may be used as the structural model, with the same grid, connection and property bulk data cards being used for both types of models. Grid point temperatures for the subsequent thermally induced deformation and stress analyses must be supplied on TEMP bulk data cards. A THERMAL (PUNCH) request in the Case Control Deck will cause the NTA to produce these required TEMP cards automatically. Detailed descriptions of the Bulk Data Deck will be given in section 3.5.

One of the three rigid formats which define the specific solution algorithms must be selected in the EXECUTIVE CONTROL DECK as will be discussed in detail in section 3.3. Certain required bulk data cards will be a function of the rigid format selected, and therefore a general description of the three rigid formats is in order:

1. Linear Steady-State Thermal Analysis – LSS thermal analysis uses APProach HEAT, SØLution 1. The rigid format is the same as that used for static structural analysis. This implies that several thermal loading conditions and constraint sets can be solved in one job, by using subcases in the Case Control Deck.

2. Nonlinear Steady-State Thermal Analysis – NLSS thermal analysis uses APProach HEAT, SØLution 3. This rigid format allows temperature-dependent conductivities of the elements, temperature-dependent convective film coefficients, nonlinear thermal radiation, and a limited use of multipoint constraints. The solution is iterative, and the user is given the option of supplying values on PARAM bulk data cards to control the solution process and to furnish required constants if the radiative mode of thermal analysis is used.

   The iterative solution requires that the user supply an estimate of the final temperature distribution vector. This estimate is used to calculate the reference conductivity plus radiation matrix needed for the iteration. The estimated temperature set is also used at all constrained (constant temperature) grid points to supply the prescribed temperatures. The values of the estimated temperatures are given on TEMP bulk data cards, and they are selected by the TEMP(MATERIAL) card in the Case Control Deck. The SPC1 bulk data card should be used to identify the constrained points.

Iteration may stop for the following reasons:

a. Normal convergency: ϵ_T < EPSHT, where ϵ_T is the per unit error estimate of the temperatures calculated.

b. Number of iterations > MAXIT.

c. Unstable: $|\lambda_1| < 1$ and the number of iterations >3, where λ_1 is a stability estimator.

d. Insufficient time (considers the CPU time used, the estimated CPU iteration time, and the time specified by the Executive Control Card TIME) to perform another iteration and output data.

The controlling parameters EPSHT (to test the convergence of the solution) and MAXIT (to limit the maximum number of iterations) are supplied by the user on PARAM Bulk Data Cards. Details are given in section 3.5.1(6). Error and stability estimates ϵ_p, λ_1 and ϵ_T for all iterations may be output by requesting DIAG 18 in the Executive Control Deck, where ϵ_p is the ratio of the Euclidian norms of the residual (error) loads to the applied thermal loads on the unconstrained grid points.

3. Transient Thermal Analysis – Transient thermal analysis uses APProach HEAT, SØLution 9 for both linear and nonlinear cases. This rigid format may include conduction, convection, nonlinear radiation, and all NASTRAN nonlinear elements. Extra points may be used to define new variables in conjunction with transfer functions. All grid points associated with nonlinear thermal loads must be in the solution set, and the condensation capability (OMIT option) for both linear and nonlinear boundary conditions achievable through matrix transfers and partitionings is currently under development. Static thermal load cards must be referenced by a TLØADi card for use in transient thermal analysis. Thermal loads are requested in case control with a DLØAD card. To obtain a prescribed temperature at a grid point, a scalar heat conduction element, e.g., CELAS2, with an arbitrary large value of conductance K_0 is connected between the grid point in question and ground. A large thermal load Q is also applied to that grid point and the desired temperature $T = Q/K_0$ is then obtained as a constant or a function of time (the latter if Q varies with time). Initial condition temperatures are specified on TEMP bulk data cards and are requested by the IC card in the Case Control. Previous steady-state or transient thermal solutions can easily be used as initial conditions for a later run, since temperature results can automatically be output as punched TEMP cards. Supplying an estimate of the final temperature distribution vector is an option for a transient thermal problem with radiation. If used, temperatures are specified on TEMP Bulk Data cards, and the TEMP set identification is requested by a TEMP (MATERIAL) card in the Case Control. Time steps controlling the process of integration are specified on the TSTEP bulk data card.

The user is also given the option of supplying values on PARAM bulk data cards to control the integration solution processing and to furnish required constants when thermal radiative interchange is involved.

3.2 General Description of Data Deck

A complete $\overline{N}$TA input deck begins with the required job control cards. The type and number of these cards will vary with the installation. Instructions for the preparation of these job control cards should be obtained from the programming staff at each installation.

The main body of the $\overline{N}$TA Data Deck consists of the following three sections:

1. Executive Control Deck
2. Case Control Deck
3. Bulk Data Deck

In some cases, the NASTRAN card may precede the Executive Control Deck. The NASTRAN card is used to change the default values for certain operational parameters, such as buffer size and machine model number. The NASTRAN card is optional, but, if present, it must be the first card of the NASTRAN Data Deck. The NASTRAN card is a free-field card (similar to cards in the Executive Control Deck). Its format is as follows:

NASTRAN keyword$_1$ = value, keyword$_2$ = value, . . .

For example

NASTRAN SYSTEM(55) = 2

SYSTEM(55) is used to specify numerical precision: 1 = single, 2 = double, the default is 1.

Additional information concerning the NASTRAN card is given in section 6.3.1 of the NASTRAN Programmer's Manual.

The Executive Control Deck begins with the NASTRAN ID card and ends with the CEND card, as shown in figure 3.2. It identifies the job and the type of solution to be performed. It also declares the general conditions under which the job is to be executed, such as, maximum time allowed, type of system diagnostics desired, restart conditions, and whether or not the job is to be checkpointed. If the job is to be executed with a rigid format, the number of the rigid format is declared along with any alterations to the rigid format that may be desired. If Direct Matrix Abstraction is used, the complete DMAP sequence must appear in the Executive Control Deck. The executive control cards and examples of their use are described in section 3.3.

The Case Control Deck begins with the first card following CEND and ends with the card preceeding BEGIN BULK. It makes selections from the Bulk Data Deck, and makes output requests for printing, punching and plotting. It also defines the subcases, if any, for the problem. A general discussion of the functions of the Case Control Deck and a detailed description of the cards used in this deck are given in section 3.4.

The Bulk Data Deck begins with the card following BEGIN BULK and ends with the card preceding ENDDATA. It contains all of the details of the idealized thermal model and initial and boundary conditions for the solution. The BEGIN BULK and ENDDATA cards must be

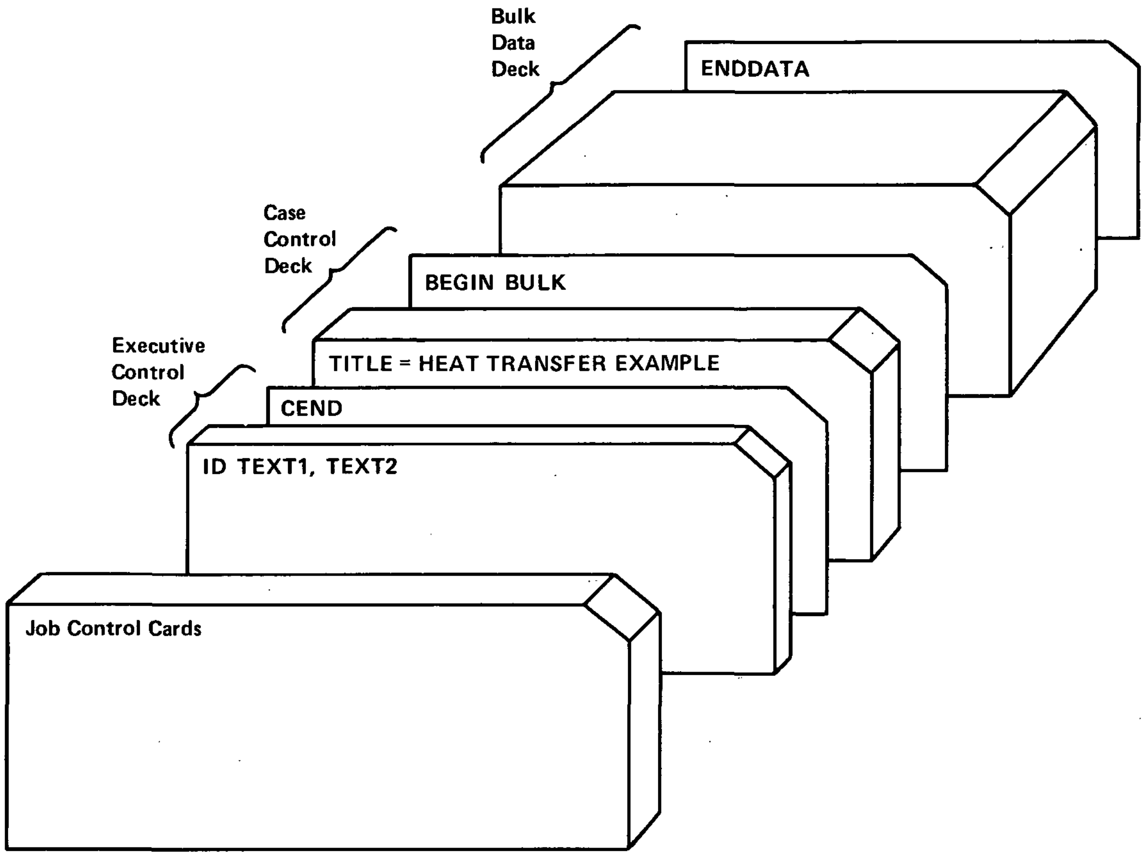


Figure 3.2. A complete NASTRAN THERMAL ANALYZER input deck.

present even though no new bulk data are being introduced into the problem, or all of the bulk data are coming from an alternate source, such as User's Master File or user-generated input. The format of the BEGIN BULK card is free field. The ENDDATA card must begin in column 1 or 2. Generally speaking only one model can be defined in the Bulk Data Deck. However, some of the bulk data, such as cards associated with thermal loading conditions, constraints, direct input matrices and transfer functions may exist in multiple sets. All types of data that are available in multiple sets are discussed in section 3.4. Only sets selected in the Case Control Deck will be used in any particular solution.

Comment cards may be inserted in any of the parts of the NASTRAN Data Deck. These cards are identified by a $ in column one. Columns 2-72 may contain any desired text.

Except for the IBM 360/370 series, all NASTRAN Thermal Analyzer data cards must be punched using the character set shown in the following table. The EBCDIC character set may be used on the IBM 360/370 series. Any EBCDIC characters are automatically translated into the required character set as follows:

Character	Card Punch(s)	Character	Card Punch(s)
blank	blank	N	11–5
0	0	Ø	11–6
1	1	P	11–7
2	2	Q	11–8
3	3	R	11–9
4	4	S	0–2
5	5	T	0–3
6	6	U	0–4
7	7	V	0–5
8	8	W	0–6
9	9	X	0–7
A	12–1	Y	0–8
B	12–2	Z	0–9
C	12–3	$	11–3–8
D	12–4	/	0–1
E	12–5	+	12
F	12–6	–	11
G	12–7	(	0–4–8
H	12–8	)	12–4–8
I	12–9	'	4–8
J	11–1	=	3–8
K	11–2	,	0–3–8
L	11–3	.	12–3–8
M	11–4	*	11–4–8

3.3 Executive Control Deck

The format of the Executive Control cards is free field. The name of the operation (e.g., CHKPNT) begins in column 1 and is separated from the operand by one or more blanks. The fields in the operand are separated by commas, and may be integers (Ki) or alphanumeric (Ai) as indicated in the following control card descriptions. The first character of an alphanumeric field must be alphabetic followed by up to 7 additional alphanumeric characters. Blank characters may be placed adjacent to separating commas if desired. The individual cards are described in section 3.3.1 and examples follow in section 3.3.2.

3.3.1 Executive Control Card Descriptions

ID A1, A2 Required.

A1, A2 – Any legal alphanumeric fields chosen by the user for problem identification.

RESTART A1, A2, K1/K2/K3, Required for Restart.

A1, A2 – Fields taken from ID card of previously checkpointed problem.

K1/K2/K3 – Month/Day/Year that Problem Tape was generated.

The complete restart dictionary consists of this card followed by one card for each file checkpointed. The restart dictionary is automatically punched when operating in the checkpoint mode. All subsequent cards are continuations of this logical card.

Each continuation card begins with a sequence number. Each type of continuation card will be documented separately.

1. Basic continuation card

   NO,DATABLØCK,FLAG=Y,REEL=Z,FILE=W

   where: NO is the sequence number of the card. The entire dictionary must be in sequence by this number.

   DATABLØCK is the name of the data block referenced by this card.

   FLAG=Y defines the status of the data block where Y = 0 is the normal case and Y = 4 implies this data block is equivalenced to another data block. In this case (FLAG=4) the file number points to a previous data block which is the "actual" copy of the data.

   REEL=Z specifies the reel number as the Problem Tape can be a multi-reel tape. Z = 1 is the normal case.

   FILE=W specifies the GINØ (internal) file number of the data block on the Problem Tape. A zero value indicates the data block is purged. For example:

   1,GPL,FLAGS=0,REEL=1,FILE=7 says data block GPL occupies file 7 of reel 1.

2,KGG,FLAGS=4,REEL=1,FILE=20 says KGG is equivalenced to the data block which occupies file 20. (Note that FLAGS=4 cards usually occur in at least pairs as the equivalenced operation is at least binary.)

3,USETD,FLAGS=0,REEL=1,FILE=0 implies USETD is purged.

2. Reentry point card:

NO,REENTER AT DMAP SEQUENCE NUMBER N

where: NO is the sequence number of the card.

N is the sequence number associated with the DMAP instruction at which the problem will restart. This value may be changed by adding a final such card (i.e., only the last such card is operative). This may be necessary when restarting from a Rigid Format to a DMAP sequence (to print a matrix for example).

There are four types of restarts: Unmodified Restart, Modified Restart, Rigid Format Switch and Pseudo Modified Restart. The function of the reentry point is different in each case. On an Unmodified Restart the program continues from the reentry point. On a Modified Restart modules which must be run to process the modified data which are ahead of the reentry point are executed first. The program then continues from the reentry point. On a Rigid Format Switch (going from a Rigid Format to another) the reentry point is meaningless in that it was determined for another DMAP sequence. In this case the data blocks available are consulted to determine the proper sequence of modules to run. A Pseudo Modified Restart (defined by the existence of only changes to output producing data such as plotter requests) is treated like a modified restart. The type of restart is implied by the changes made in the NTA Data Deck. No explicit request for a particular kind of restart is required. See section 3.1 of the NASTRAN User's Manual for additional information.

3. End of dictionary card:

$ END ØF CHECKPØINT DICTIØNARY

This card is simply a comment card but is punched to signal the end of the dictionary for user convenience. The program does not need such a card. Terminations associated with non-NASTRAN failures (operator intervention, maximum time, etc.) will not have such a card punched.

UMF K1, K2 Required when using User's Master File.

K1 – User specified tape identification number assigned during the generation of the User's Master File.

K2 – Problem identification number assigned during generation of User's Master File.

CHKPNT A1 or CHKPNT A1, A2 Optional.

A1 – YES if problem is to be checkpointed, NØ if problem is not to be checkpointed – default is NØ.

A2 – DISK if checkpoint file is on direct access device. If the DISK option is used, the user must instruct the resident operating system to permanently catalog the checkpoint file.

APP A Required.

A – HEAT indicates one of the heat transfer rigid formats.

A – DMAP indicates Direct Matrix Abstraction Approach (DMAP).

SØL K Required when using rigid format (see section 3.1 for available options)

K – Solution number of Rigid Format (K=1,3, or 9).

ALTER K1, K2 Optional.

K1, K2 – First and last DMAP instructions of series to be deleted and replaced with any following DMAP instructions.

ALTER K Optional.

K – Input any following DMAP instructions after statement K.

TIME K Required.

K – Maximum allowable execution time in minutes.

ENDALTER Required when using ALTER.

Indicates end of DMAP alterations.

DIAG K Optional request for diagnostic output.

K = 1	Dump memory when non-preface fatal message is generated.
K = 2	Print File Allocation Table (FIAT) following each call to the File Allocator.
K = 3	Print status of the Data Pool Dictionary (DPD) following each call to the Data Pool Housekeeper.
K = 4	Print the Operation Sequence Control Array (ØSCAR).
K = 5	Print BEGIN time on-line for each functional module.
K = 6	Print END time on-line for each functional module.
K = 7	Print eigenvalue extraction diagnostics for real and complex determinant methods.
K = 8	Print matrix trailers as the matrices are generated.
K = 9	Suppress echo of checkpoint dictionary.
K = 10	Use alternate nonlinear loading in TRD. (Replace $\{N_{n+1}\}$ by 1/3 $\{N_{n+1} + N_n + N_{n-1}\}$)

K = 11	Print all active row and column possibilities for decomposition algorithms.
K = 12	Print eigenvalue extraction diagnostics for complex inverse power.
K = 13	Print open core length.
K = 14	Print the Rigid Format (NASTRAN SØURCE PRØGRAM CØMPILATIØN) for all non-Restart runs.
K = 15	Trace GINØ ØPEN/CLØSE operations.
K = 16	Trace real inverse power eigenvalue extraction operations.
K = 17	Punch the DMAP sequence that is compiled.
K = 18	Trace Heat Transfer iterations.
K = 19	Print data for MPYAD method selection.
K = 20	Generate de-bug printout (for NASTRAN programmers who include CALL BUG in their subroutines).
K = 21	Print GP4 set definition.
K = 22	Print GP4 degree of freedom definition.
K = 23–26	Not used.
K = 27	Input File Processor (IFP) table dump.
K = 28	Punch the link specification table (Deck XBSBD).
K = 29	Process link specification table update deck.
K = 30	Punch alters to the XSEMi decks (i set via DIAG 1–15).
K = 31	Print link specification table and module properties list data.

Multiple options may be selected by using multiple integers separated by commas. Other options and other rules associated with the DIAG card which primarily concern the programmer can be found in section 6.11.3 of the NASTRAN Programmer's Manual.

BEGIN$ Required when using DMAP approach.

Indicates beginning of DMAP sequence. This card is supplied as part of a Rigid Format.

END$ Required when using DMAP approach.

Indicates end of DMAP sequence. This card is supplied as part of a Rigid Format.

UMFEDIT Required when using User's Master File Editor (see section 2.5 of the NASTRAN User's Manual).

CEND Required

Indicates end of Executive Control cards.

The ID card must appear first and CEND must be the last card of the Executive Control Deck. Otherwise the Executive Control card groups (RESTART dictionary, DMAP sequence, ALTER packet) can be in any order.

3.3.2 *Executive Control Deck Examples*

1. Cold start, no checkpoint, rigid format, diagnostic output.

```
ID        MYNAME,PROBLEMID
APP       HEAT
SØL       1
TIME      5
DIAG      21, 22
CEND
```

2. Cold start, checkpoint, rigid format.

```
ID        PERSØNZZ, SPACECFT
CHKPNT    YES
APP       HEAT
SØL       1
TIME      15
CEND
```

3. Restart, no checkpoint, rigid format. The restart dictionary indicated by the brace is automatically punched on previous run in which the CHKPNT option was selected by the user.

```
ID JØESHMØE, PRØJECTX
RESTART PERSØNZZ, SPACECFT, 05/13/67,
   1, XVPS, FLAGS=0, REEL=1, FILE=6
   2, REENTER AT DMAP SEQUENCE NUMBER 7
   3, GPL, FLAGS=0 REEL=1, FILE=7
                    .
                    .
                    .
$ END OF CHECKPØINT DICTIØNARY
APP       HEAT
SØL       3
TIME      10
CEND
```

4. Cold start, no checkpoint, DMAP. User-written DMAP program is indicated by braces.

```
ID        IAM007,TRYIT
APP       DMAP
BEGIN $

{DMAP statements go here}
```

```
END    $
TIME        8
CEND
```

5. Restart, checkpoint, altered rigid format, diagnostic output.

```
ID GØØDGUY, NEATDEAL
RESTART BADGUY, NØSHØW, 18/09/75
   1, XVPS, FLAGS=0, REEL=1, FILE=6
   2, REENTER AT DMAP SEQUENCE NUMBER 7
   3, GPL, FLAGS=0, REEL=1, FILE=7
                      .
                      .
                      .
$ END ØF CHECKPØINT DICTIØNARY
CHKPNT      YES
DIAG        2,4
APP         HEAT
SØL         3
TIME        15
ALTER       20
MATPRN      KGGX,,,,// $
TABPT       GPST,,,,// $
ENDALTER
CEND
```

3.4 Case Control Deck

3.4.1 Subcase Definition

For the user's convenience, a provision has been made for the LSS thermal problems to permit a separate subcase to be defined for each thermal loading condition and also for each set of temperature constraints. This capability is currently being extended for use in the solution of NLSS problems.

The Case Control Deck is structured so that a minimum amount of repetition is required. Only one level of subcase definition is provided. All items placed above the subcase level (ahead of the first subcase) will be used for all following subcases, unless overridden within the individual subcase.

Provision has also been made for the combination of the results of several subcases. This is convenient for studying various combinations of individual thermal loading conditions and for the superposition of solutions for symmetrical and antisymmetrical boundaries.

The following Case Control cards commonly used in thermal analysis are associated with subcase definition:

1. SUBCASE – defines the beginning of a subcase that is terminated by the next subcase delimiters encountered.

2. SUBCØM – defines a combination of two or more preceding subcases in LSS problems. Output requests above the subcase level are used.

3. SUBSEQ – must appear in a subcase defined by SUBCØM to give the coefficients for making the linear combination of the preceding subcases.

The following examples of Case Control Decks indicate typical ways of defining subcases:

1. Linear steady-state thermal analysis with multiple loads.

```
ØUTPUT
  TEMPERATURE = ALL
MPC = 3
  SUBCASE 1
    SPC = 2
    TEMPERATURE(MATERIAL) = 101
    LØAD = 11
  SUBCASE 2
    SPC = 2
    TEMPERATURE(MATERIAL) = 101
    LØAD = 12
  SUBCASE 3
    SPC = 4
    TEMPERATURE(MATERIAL) = 102
```

```
SUBCASE 4
  MPC = 4
  SPC = 4
```

Four subcases are defined in this example. The temperatures at all grid points will be printed for all four subcases. MPC = 3 will be used for the first three subcases and will be overridden by MPC = 4 in the last subcase. Since the constraints for prescribed temperatures are the same for subcases 1 and 2 and the subcases are contiguous, the steady-state solutions will be performed simultaneously. In subcase 4 the static loading will result entirely from prescribed temperatures of grid points.

2. Linear combination of subcases.

```
SPC = 2
ØUTPUT
  SET 1 = 1 THRU 10,20,30
  TEMPERATURE = ALL
  ELFØRCE = 1
SUBCASE 1
  LØAD = 101
  ØLØAD = ALL
SUBCASE 2
  LØAD = 201
  ØLØAD = ALL
SUBCØM 51
  SUBSEQ = 1.0,1.0
SUBCØM 52
  SUBSEQ = 2.5,1.5
```

Two static thermal loading conditions are defined in subcases 1 and 2. SUBCØM 51 defines the sum of subcases 1 and 2. SUBCØM 52 defines a linear combination consisting of 2.5 times subcase 1 plus 1.5 times subcase 2. The temperatures at all grid points and the heat flows and gradients for the element numbers in SET 1 will be printed for all four subcases. In addition, the nonzero components of the static thermal load vectors will be printed for subcases 1 and 2.

3.4.2 Data Selection

The Case Control cards that are used for selecting items from the Bulk Data Deck are listed below in functional groups. A detailed description of each card is given in section 3.4.4. The first four characters of the mnemonic are sufficient if unique.

The following Case Control cards are associated with the selection of applied thermal loads for both steady-state and transient thermal analyses:

1. <u>DLØAD</u> – selects time-varying thermal loading condition.

2. LØAD – selects static loading condition.

3. NØNLINEAR – selects nonlinear loading condition for transient thermal analysis.

The following Case Control cards are used for the selection of constraints:

1. AXISYMMETRIC – selects boundary conditions for conical shell elements.

2. MPC – selects set of multipoint constraints.

3. SPC – selects set of single-point constraints.

The following Case Control cards are used for the selection of direct input matrices:

1. B2PP – selects direct input thermal capacitance matrices.

2. K2PP – selects direct input thermal conductance matrices.

3. TFL – selects transfer functions.

The following Case Control cards specify the conditions for transient thermal analyses:

1. IC – selects the initial condition for transient thermal analysis.

2. TSTEP – selects time steps to be used for integration in transient thermal analysis.

The following Case Control card is used to provide an estimated temperature distribution vector:

TEMPERATURE(MATERIAL) – selects an estimated temperature distribution vector to be used for solutions using SØL 3 (mandatory) and SØL 9 (optional).

3.4.3 Output Selection

Printer output requests may be grouped in packets following ØUTPUT cards or the individual requests may be placed anywhere in the Case Control Deck ahead of any structure plotter or curve plotter requests. Plotter requests are described in section 4 of the NASTRAN User's Manual. The Case Control cards that are used for output selection are listed below in functional groups. A detailed description of each card is given in section 3.4.4.

The following cards are associated with output control, titling and bulk data echoes:

1. TITLE – defines a text to be printed on first line of each page of output.

2. SUBTITLE – defines a text to be printed on second line of each page of output.

3. LABEL – defines a text to be printed on third line of each page of output.

4. LINE – sets the number of data lines per printed page, default is 50 for 11-inch paper.

5. MAXLINES – sets the maximum number of output lines, default is 20,000.

6. ECHØ – selects echo options for Bulk Data Deck, default is a sorted bulk data echo.

The following cards are used in connection with some of the specific output requests for calculated quantities:

1. SET – defines lists of grid point numbers, element numbers or output times for use in output requests.

2. TSTEP – references a bulk data card TSTEP which defines, among other quantities, the value N, where transient output will be produced for every Nth time step.

The following cards are used to make output requests for different thermal quantities:

1. ELFØRCE – requests the heat flows and gradients for a set of selected elements.

2. SPCFØRCES – requests the power required to sustain the prescribed temperatures at those single-point constrained grid points in the selected output set.

3. THERMAL – requests the temperatures for a selected set of grid, scalar, and extra points.

4. ØLØAD – requests the applied linear thermal loads for a selected set of grid, scalar, and extra points.

5. NLLØAD – requests the applied nonlinear thermal loads for a selected set of grid, scalar, and extra points.

6. VELØCITY – requests the time rate change of temperature for a selected set of grid, scalar, and extra points.

3.4.4 Case Control Card Descriptions

The format of the Case Control cards is free-field. In presenting general formats for each card embodying all options, the following conventions are used:

1. Upper-case letters must be punched as shown.

2. Lower-case letters indicate that a substitution must be made.

3. Braces { } indicate that a choice of contents is mandatory.

4. Brackets [ ] contain an option that may be omitted or included by the user.

5. Underlined options or values are the default values.

6. Physical card consists of information punched in columns 1 through 72 of a card. Most Case Control cards are limited to a single physical card.

7. Logical card may have more than 72 columns with the use of continuation cards.

The heat conduction element (structure) plotter output request packet and the x–y output request packet, while part of the Case Control Deck, are treated separately. They are discussed in sections 4.2 and 4.3, respectively, of the NASTRAN User's Manual.

A list of Case Control cards commonly used in thermal modeling is summarized in alphabetical order as follows: B2PP, DLØAD, ECHØ, ELFØRCE, IC, K2PP, LABEL, LINE, LØAD, MAXLINES, MPC, NLLØAD, NØNLINEAR, ØTIME, ØLOAD, ØUTPUT, SET, SPC, SPCFØRCES, SUBTITLE, THERMAL, TFL, TEMPERATURE, TITLE, TSTEP, VELØCITY.

• Case Control Data Card B2PP – Direct Input Thermal Capacitance (Damping) Matrix Selection.

Description: Selects a direct input thermal capacitance (damping) matrix.

Format and Example(s):

B2PP = name

B2PP = BDMIG

B2PP = B2PP

Option	Meaning
name	BCD name of $[B^2_{pp}]$ matrix that is input on the DMIG or DMIAX bulk data card.

Remarks:

1. B2PP is used only in transient thermal problems.
2. DMIG and DMIAX matrices will not be used unless selected.

• Case Control Data Card DLØAD – Time-dependent Load Set Selection.

Description: Selects the time-dependent (dynamic) load to be applied in a Transient problem.

Format and Example(s):

DLØAD = n

DLØAD = 73

Option	Meaning
n	Set identification of a DLØAD, TLØAD1, or TLØAD2 card (Integer > 0).

Remarks:

1. The above loads will not be used by the $\overline{\text{N}}$TA unless selected in Case Control.
2. TLØAD1 and TLØAD2 may only be selected in a Transient problem.

• Case Control Data Card ECHØ – Bulk Data Echo Request.

Description: Requests echo of Bulk Data Deck.

Format and Example(s):

$$\text{ECHØ} = \left\{ \begin{array}{l} \underline{\text{SØRT}} \\ \text{UNSØRT} \\ \text{BØTH} \\ \text{NØNE} \end{array} \right\}$$

ECHØ = BØTH

ECHØ = SØRT, UNSØRT

Option	Meaning
SØRT	Sorted echo will be printed.
UNSØRT	Unsorted echo will be printed.
BØTH	Both sorted and unsorted echo will be printed.
NØNE	No echo will be printed.

Remarks:

1. If no ECHØ card appears a sorted echo will be printed.
2. If CHKPNT = YES a sorted echo will be printed unless ECHØ = NØNE.

• Case Control Data Card ELFØRCE – Element Heat Flow and Gradient Output Request.

Description: Requests form and type of element heat flow output.

Format and Example(s):

$$\text{ELFØRCE} \quad \left[\left(\begin{array}{c}\underline{\text{SØRT1}}\\ \text{SØRT2}\end{array}, \begin{array}{c}\underline{\text{PRINT}}\\ \text{PUNCH}\end{array}\right)\right] = \left\{\begin{array}{c}\text{ALL}\\ n\\ \text{NØNE}\end{array}\right\}$$

ELFØRCE = ALL

ELFØRCE(PUNCH, PRINT) = 17

ELFØRCE = 25

Option	Meaning
SØRT1	Output will be presented as a tabular listing of elements with associated heat flows and gradients. SØRT1 is not available on Transient problems (where the default is SØRT2).
SØRT2	Output will be presented as a tabular listing of time for each element type. SØRT2 is available only in Transient problems, where SORT1 is unavailable without a DMAP alter.
PRINT	The printer will be the output media.
PUNCH	The card punch will be the output media.
ALL	Heat flows for all elements will be output.
NØNE	Heat flows for no elements will be output.
n	Set identification of a previously appearing SET card. Only heat flows of elements whose identification numbers appear on this SET card will be output (Integer > 0).

Remarks:

1. Both PRINT and PUNCH may be requested.
2. ALL cannot be used in a Transient problem.
3. FØRCE is an alternate form and is entirely equivalent to ELFØRCE.
4. ELFØRCE = NØNE allows overriding an overall request.

- Case Control Data Card IC – Transient Initial Condition Set Selection.

Description: To select the initial conditions for Transient problems.

Format and Example(s):

IC = n

IC = 17

Option	Meaning
n	Set identification of TEMP and/or TEMPD cards (Integer > 0). TIC cards may also be referenced though this is not a preferred technique.

Remarks:

Initial conditions will all be zero unless IC is used in the Case Control.

• Case Control Data Card K2PP – Direct Input Thermal Conductance Matrix Selection.

Description: Selects a direct input thermal conductance matrix.

Format and Example(s):

K2PP = name

K2PP = KDMIG

K2PP = K2PP

Option	Meaning
name	BCD name of a DMIG matrix that is input on the bulk data card DMIG.

Remarks:

1. K2PP is used only in transient thermal problems.
2. DMIG matrices will not be used unless selected.

• Case Control Data Card LABEL – Output Label.

Description: Defines a BCD label which will appear on the third heading line of each page of NASTRAN printer output.

Format and Example(s):

LABEL = {Any BCD data}

LABEL = Radiator of ØSØ-I CUE Unit

Remarks:

1. LABEL appearing at the subcase level will label output for that subcase only.
2. LABEL appearing before all subcases will label any outputs which are not subcase dependent.
3. If no LABEL card is supplied, the label line will be blank.
4. LABEL information is also placed on NASTRAN plotter output as applicable.

• Case Control Data Card LINE – Data Lines Per Page.

Description: Defines the number of data lines per printed page.

Format and Example(s):

$$\text{LINE} = \begin{Bmatrix} 50 \\ \hline n \end{Bmatrix}$$

LINE = 35

Option	Meaning
n	Number of data lines per page (Integer > 0).

Remarks:

1. If no LINE card appears, 50 is used.
2. For 11 inch paper, 50 is recommended; for 8-1/2 inch paper, 35 is recommended.

• Case Control Data Card LØAD – Static Thermal Load Set Selection.

Description: Selects the static thermal load set to be applied to the thermal model.

Format and Example(s):

LØAD = n

LØAD = 15

Option	Meaning
n	Set identification of at least one thermal load card and hence must appear on at least one SLØAD, QVECT, QVØL, QBDY1, QBDY2, QHBDY Card (Integer > 0).

Remarks:

1. The above static load cards will not be used by the NTA unless selected in Case Control.
2. The total load applied will be the sum of external (LØAD), and constrained (SPC) loads.

• Case Control Data Card MAXLINES – Maximum Number of Output Lines.

Description: Sets the maximum number of output lines to a given value.

Format and Example(s):

$$\text{MAXLINES} = \begin{Bmatrix} 20000 \\ \hline n \end{Bmatrix}$$

MAXLINES = 50000

Option	Meaning
n	Maximum number of output lines which the user wishes to allow (Integer > 0).

Remarks:

1. Any time this number is exceeded, NASTRAN will terminate through PEXIT.
2. This does not override any system MAXLINES parameters such as those on JØB cards or space requests.

• Case Control Data Card MPC – Multipoint Constraint Set Selection.

Description: Selects the multipoint constraint set to be applied to the thermal model.

Format and Example(s):

MPC = n

MPC = 17

Option	Meaning
n	Set identification of a Multipoint-Constraint Set and hence must appear on at least one MPC or MPCADD card (Integer > 0).

Remarks:

MPC or MPCADD cards will not be used by NASTRAN unless selected in Case Control.

• Case Control Data Card NLLØAD – Nonlinear Load Output Request.

Description: Requests form and type of nonlinear load output for transient thermal problems.

Format and Example(s):

$$\text{NLL}\varnothing\text{AD}\ \left[\left(\frac{\text{PRINT}}{\text{PUNCH}}\right)\right] = \left\{\begin{array}{c}\text{ALL}\\ \text{n}\\ \text{N}\varnothing\text{NE}\end{array}\right\}$$

NLLØAD = ALL

Option	Meaning
PRINT	The printer will be the output media.
PUNCH	The card punch will be the output media.
ALL	Nonlinear loads for all solution points will be output.

Option	Meaning
NØNE	Nonlinear loads will not be output.
n	Set identification of previously appearing SET card (Integer > 0). Only nonlinear loads for points whose identification numbers appear on this SET card will be output.

Remarks:

1. Nonlinear loads are output only in the solution (D or H) set.
2. The output will have a SØRT2 format.
3. Both PRINT and PUNCH may be used.
4. NLLØAD = NØNE allows overriding an overall output request.

• Case Control Data Card NØNLINEAR – Nonlinear Load Set Selection.

Description: Selects nonlinear load for transient thermal problems.

Format and Example(s):

NØNLINEAR = n

NØNLINEAR LØAD SET = 75

Option	Meaning
n	Set identification of NØLINi cards (Integer > 0).

Remarks:

NØLINi cards will not be used unless selected in Case Control.

• Case Control Data Card ØTIME – Output Time Step Set.

Description: Selects from the solution set of time steps a subset for output requests.

Format and Example(s):

$$ØTIME = \left\{\frac{\text{ALL}}{\text{n}}\right\}$$

ØTIME = ALL

ØTIME SET = 15

Option	Meaning
ALL	Output for all time steps will be printed out.
n	Set identification of previously appearing SET card (Integer > 0). Output for time steps closest to those given on this SET card will be output.

Remarks:

OTIME overrides the output data on a selected TSTEP card.

• Case Control Data Card ØLØAD – Applied Thermal Load Output Request.

Description: Requests form and type of applied thermal load vector output.

Format and Example(s):

$$\text{ØLØAD} \left[\left(\frac{\text{SØRT1}}{\text{SØRT2}}, \frac{\text{PRINT}}{\text{PUNCH}}\right)\right] = \left\{\begin{matrix}\text{ALL}\\ \text{n}\\ \text{NØNE}\end{matrix}\right\}$$

ØLØAD = ALL

Option	Meaning
SØRT1	Output will be presented as a tabular listing of grid points with associated thermal loads. SØRT1 is not available on Transient problems (where the default is SØRT2).
SØRT2	Output will be presented as a tabular listing of time for each grid point. SØRT2 is available only in Transient problems, where SORT1 is unavailable without a DMAP alter.
PRINT	The printer will be the output media.
PUNCH	The card punch will be the output media.
ALL	Applied thermal loads for all points will be output. (SØRT1 will only output nonzero values.)
NØNE	Applied thermal loads for no points will be output.
n	Set identification of previously appearing SET card. Only loads on points whose identification numbers appear on this SET card will be output (Integer > 0).

Remarks:

1. Both PRINT and PUNCH may be requested.

2. In a steady-state problem a request for SØRT2 causes thermal loads at all points (zero and nonzero) to be output in SØRT1 format.

3. ØLØAD = NØNE allows overriding an overall output request.

• Case Control Data Card ØUTPUT – Output Packet Delimiter.

Description: Delimits the various output packets, structure plotter, curve plotter, and printer/punch.

Format and Example(s):

$$\text{ØUTPUT} \left[\begin{pmatrix} \text{PLØT} \\ \text{XYØUT} \\ \text{XYPLØT} \end{pmatrix} \right]$$

ØUTPUT

ØUTPUT(PLØT)

ØUTPUT(XYØUT)

Option	Meaning
No qualifier	Beginning of printer output packet – this is not a required card.
PLØT	Beginning of structure plotter packet. This card must precede all structure plotter control cards.
XYØUT or XYPLØT	Beginning of curve plotter packet. This card must precede all curve plotter control cards. XYPLØT and XYØUT are entirely equivalent.

Remarks:

1. The structure plotter packet and the curve plotter packet must be at the end of the Case Control Deck. Either may come first.

2. The delimiting of a printer packet is completely optional.

• Case Control Data Card SET – Set Definition Card.

Description:

1. Lists identification numbers (point or element) for output requests.

2. Lists the times at which output will be provided during a transient run.

Format and Example(s):

1. SET n = $\{i_1 [,i_2, i_3 \text{ THRU } i_4 \text{ EXCEPT } i_5, i_6, i_7, i_8 \text{ THRU } i_9]\}$

   SET 77 = 5

   SET 88 = 5, 6, 7, 8, 9, 10 THRU 55 EXCEPT 15, 16, 77, 78, 79, 100 THRU 300

   SET 99 = 1 THRU 100000

2. SET n = $\{r_1 [, r_2, r_3, r_4]\}$

   SET 101 = 1.0, 2.0, 3.0

   SET 105 = 1.009, 10.2, 13.4, 14.0, 15.0

Option	Meaning
n	Set identification (Integer > 0). Any set may be redefined by reassigning its identification number. Sets inside SUBCASE delimiters are local to the SUBCASE.
i_1, i_2 etc.	Element or point identification number at which output is requested (Integer > 0). If no such identification number exists, the request is ignored.
i_3 THRU i_4	Output at set identification numbers i_3 thru i_4 ($i_4 > i_3$).
EXCEPT	Set identification numbers following EXCEPT will be deleted from output list as long as they are in the range of the set defined by the immediately preceding THRU.
r_1, r_2 etc.	Times for transient output (Real > 0.0). The nearest solution time will be output. EXCEPT and THRU cannot be used.

Remarks:

A SET card may be more than one physical card. A comma (,) at the end of a physical card signifies a continuation card. Commas may not end a set.

• Case Control Data Card SPC – Single-Point Constraint Set Selection.

Description: Selects the single-point constraint set to be applied to the thermal model.

Format and Example(s):

SPC = n

SPC = 10

Option	Meaning
n	Set identification of a single-point constraint set and hence must appear on a SPC, SPC1 or SPCADD card (Integer > 0).

Remarks:

SPC, SPC1 or SPCADD cards will not be used by the $\overline{\text{N}}$TA unless selected in Case Control.

- Case Control Data Card SPCFØRCES – Single-Point Constraint Thermal Forces Output Request.

Description: Requests form and type of powers required to sustain the prescribed temperatures at the single-point constrained points.

Format and Example(s):

$$\text{SPCFØRCES} \left[\left(\frac{\text{SØRT1}}{\text{SØRT2}}, \frac{\text{PRINT}}{\text{PUNCH}}\right)\right] = \begin{Bmatrix} \text{ALL} \\ \text{n} \\ \text{NØNE} \end{Bmatrix}$$

SPCFØRCES = 5

SPCFØRCES(SØRT2, PUNCH, PRINT, IMAG) = ALL

Option	Meaning
SØRT1	Output will be presented as a tabular listing of grid points with associated single-point constraint thermal forces. SØRT1 is not available on Transient problems (where the default is SØRT2).
SØRT2	Output will be presented as a tabular listing of time for each grid point. SØRT2 is available only in Transient problems, where SØRT1 is unavailable without a DMAP alter.
PRINT	The printer will be the output media.
PUNCH	The card punch will be the output media.
ALL	Single-point forces of constraint for all points will be output. (SØRT1 will only output nonzero values.)
NØNE	Single-point forces of constraint for no points will be output.
n	Set identification of previously appearing SET card. Only single-point thermal forces of constraint for points whose identification numbers appear on this SET card will be output (Integer > 0).

Remarks:

1. Both PRINT and PUNCH may be requested.
2. In a steady-state problem a request for SØRT2 causes thermal loads at all points (zero and non-zero) to be output in SØRT1 format.
3. SPCFØRCES = NØNE allows overriding an overall output request.

• Case Control Data Card SUBTITLE – Output Subtitle.

Description: Defines a BCD subtitle which will appear on the second heading line of each page of NTA printer output.

Format and Example(s):

SUBTITLE = {Any BCD data}

SUBTITLE = NØNLINEAR STEADY-STATE PRØBLEM NØ. 5

Remarks:

1. SUBTITLE appearing at the subcase level will title output for that subcase only.
2. SUBTITLE appearing before all subcases will title any outputs which are not subcase dependent.
3. If no SUBTITLE card is supplied, the subtitle line will be blank.
4. SUBTITLE information is also placed on NASTRAN plotter output as applicable.

• Case Control Data Card THERMAL – Temperature Output Request.

Description: Requests form and type of temperature vector output.

Format and Example(s):

$$\text{THERMAL} \left[\left(\frac{\text{PRINT}}{\text{PUNCH}}\right)\right] = \begin{Bmatrix} \text{ALL} \\ n \\ \text{NØNE} \end{Bmatrix}$$

THERMAL = 5

THER(PRINT,PUNCH) = ALL

Option	Meaning
PRINT	The printer will be the output media.
PUNCH	The card punch will be the output media.
ALL	Temperatures for all points will be output.
NØNE	Temperatures for no points will be output.
n	Set identification of previously appearing SET card. Only temperatures of points whose identification numbers appear on this SET card will be output (Integer > 0).

Remarks:

1. The printed output will have temperature headings and the punched output will be double field TEMP* bulk data cards. The SID on a bulk data card will be the subcase number (= 1 if no defined subcases).
2. Both PRINT and PUNCH may be requested.

• Case Control Data Card TFL – Transfer Function Set Selection.

Description: Selects the transfer function set to be added to the direct input matrices.

Format and Example(s):

TFL = n

TFL = 77

Option	Meaning
n	Set identification of a TF card (Integer > 0).

Remarks:

1. Transfer functions will not be used unless selected in the Case Control Deck.
2. Transfer functions are allowed in transient thermal problems only.
3. Transfer functions are simply another form of direct matrix input.

• Case Control Data Card TEMPERATURE – Thermal Properties Set Selection.

Description: Selects the temperature set to be used in material property calculation and/or as the estimated final temperature vector.

Format and Example(s):

TEMPERATURE(MATERIAL) = n

TEMPERATURE(MATERIAL) = 7

Option	Meaning
MATERIAL	The selected temperature table will be used to determine temperature-dependent material properties indicated on the MATTi type cards, and/or the temperature table provides an estimated final temperature vector for the solution algorithm.
n	Set identification of TEMP and/or TEMPD cards (Integer > 0).

Remarks:

1. Only one temperature-dependent material request may be made in any problem.
2. Temperature-dependent materials may not be used in transient thermal problems.
3. For nonlinear steady-state problems, an estimated final temperature set must be selected.
4. For nonlinear transient problems, an estimated final temperature vector may be selected.

• Case Control Data Card TITLE – Output Title.

Description: Defines a BCD title which will appear on the first heading line of each page of NTA.

Format and Example(s):

TITLE = {Any BCD data}

TITLE = **$// ABCDEFGHI .... $

Remarks:

1. TITLE appearing at the subcase level will title output for that subcase only.
2. TITLE appearing before all subcases will title any outputs which are not subcase dependent.
3. If no TITLE card is supplied, the title line will contain data and page numbers only.
4. TITLE information is also placed on NASTRAN plotter output as applicable.

• Case Control Data Card TSTEP – Transient Time Step Set Selection.

Description: Selects integration and output time steps for Transient problems.

Format and Example(s):

TSTEP = n

TSTEP = 731

Option	Meaning
n	Set identification of a selected TSTEP bulk data card (Integer > 0).

Remarks:

1. A TSTEP card must be selected to execute a Transient problem.
2. Only one TSTEP card may have this value of n.
3. The output time steps designated by the TSTEP card may be overridden by the ØTIME Case Control card.

- Case Control Data Card VELØCITY – Time Rate Change of Temperature Output Request.

Description: Requests time rate change of temperature output.

Format and Example(s):

$$\text{VEL}\emptyset\text{CITY} \left[\left(\frac{\text{S}\emptyset\text{RT1}}{\text{S}\emptyset\text{RT2}}, \frac{\text{PRINT}}{\text{PUNCH}}\right)\right] = \begin{Bmatrix} \text{ALL} \\ \text{n} \\ \text{N}\emptyset\text{NE} \end{Bmatrix}$$

VELØCITY = 5

VELØCITY(SØRT2, PHASE, PUNCH) = ALL

Option	Meaning
SØRT1	Output will be presented as a tabular listing of grid points with their associated temperature change rates. SØRT1 is not available on Transient problems (where the default is SØRT2).
SØRT2	Output will be presented as a tabular listing of time for each grid point. SØRT2 is available only in Transient problems, where SORT1 is unavailable without a DMAP alter.
PRINT	The printer will be the output media.
PUNCH	The card punch will be the output media.
ALL	Velocity for all solution points will be output.

Option	Meaning
NØNE	Velocity for no solution points will be output.
n	Set identification of a previously appearing SET card. Only velocities of points whose identification numbers appear on this SET card will be output (Integer > 0).

Remarks:

1. Both PRINT and PUNCH may be requested.
2. VELØCITY = NØNE allows overriding an overall output request.

3.5 Bulk Data Deck

The Bulk Data cards are used to define the thermal model and various pools of data which may be selected by Case Control at execution time.

The Bulk Data Deck may contain several thousand cards. In order to minimize the handling of large numbers of cards, provision has been made in the NTA to store the bulk data on the Problem Tape, from which it may be modified on subsequent runs. A User's Master File (section 2.5 of the NASTRAN User's Manual) is also provided for the storage of Bulk Data Decks.

For any cold start, the entire Bulk Data Deck must be submitted. Thereafter, if the original run was checkpointed, the Bulk Data Deck exists on the Problem Tape in sorted form where it may be modified and reused on restart. On restart the Bulk Data cards contained in the Bulk Data Deck are added to the bulk data contained on the Old Problem Tape. Cards are removed from the Old Problem Tape (or the User's Master File) by the use of a delete card. Cards to be deleted are indicated by inserting a Bulk Data card with a / in column one and the sorted bulk data sequence numbers in fields two and three. All Bulk Data cards in the range of the sequence numbers in fields two and three will be deleted. In the case where only a single card is deleted, field three may be left blank.

The Bulk Data Deck may be submitted with the cards in any order as a sort is performed prior to the execution of the Input File Processor. However, the machine time to perform this sorting is minimized for a deck that is already sorted. The sort time for a badly sorted deck will become significant for large decks. The user may obtain a printed copy of either the unsorted or sorted bulk data by selection in the Case Control Deck. A sorted echo is necessary in order to make modifications on a secondary execution using the Problem Tape. This echo is automatically provided unless specifically suppressed by the user.

3.5.1 Functional Description of Bulk Data Cards

1. GRID POINTS

   a. Grid Point Definition – Geometric grid points are defined on GRID Bulk Data cards by specifying their coordinates in either the basic or a local coordinate system. The implicitly defined basic coordinate system is rectangular, except when using axisymmetric elements. Local coordinate systems may be rectangular, cylindrical, or spherical. Each local system must be related directly or indirectly to the basic coordinate system. The CØRD1C, CØRD1R and CØRD1S cards are used to define cylindrical, rectangular and spherical local coordinate systems, respectively, in terms of three geometric grid points which have been previously defined. The CØRD2C, CØRD2R and CØRD2S cards are used to define cylindrical, rectangular and spherical local coordinate systems, respectively, in terms of the coordinates of three points in a previously defined coordinate system.

Six rectangular displacement components (3 translations and 3 rotations) were originally defined at each grid point for the purpose of structural analysis but only the first component is used to represent the temperature variable in the NASTRAN Thermal Analyzer. The collection of all coordinate systems in a problem is known as the global coordinate system. All matrices are formed and all thermal quantities are output in the global coordinate system.

Provision is also made on the GRID card to apply a single-point constraint, though the use of this feature is not recommended.

The GRDSET card is provided to avoid the necessity of repeating the specification of location coordinate systems and single-point constraints, when all, or many, of the GRID cards have the same entries for these items. When any of these items are specified on the GRDSET card, the entries are used to replace blank fields on the GRID card for these items.

Scalar points are defined either on an SPØINT card or by reference on a connection card for a scalar element. SPØINT cards are used primarily to define scalar points appearing in constraint equations, but to which no heat conduction elements are connected. A scalar point is implicitly defined if it is used as a connection point for any scalar element. Special scalar points, called "extra points," may be introduced for transient thermal analyses. Extra points are used in connection with transfer functions and other forms of direct matrix input used in transient thermal analyses and are defined on EPØINT cards.

b. Grid Point Sequencing – The best solution times are obtained if the grid points can be sequenced in such a manner as to create thermal conductance matrices having relatively narrow bands. In some cases the bandwidth can be substantially reduced by purposely sequencing a few of the grid points well outside the band. The resulting nonzero terms outside the band are treated individually by the triangular decomposition routines. Columns of a matrix containing nonzero terms outside the band are referred to as "active columns." The details of the partially banded decomposition routines are given in section 2.2 of the NASTRAN Theoretical Manual. If the bandwidth is large enough to cause excessive use of secondary storage devices (spill) during the triangular decomposition of the thermal conductance (stiffness) matrix in steady-state thermal (static) analysis, it may be more efficient to use the partitioning procedure described in section 1.4.4 of the NASTRAN User's Manual.

Excluding grid points that are purposely sequenced outside the band, the bandwidths of conductance matrices are proportional to the maximum difference between any two connected grid point sequence numbers. The discussion and examples that follow will discuss bandwidths and active columns in terms of geometric grid points. The semiband is defined as the maximum number of columns included from the diagonal term in any row to the most

remote term inside the band. If the diagonal terms are excluded, the semiband is proportional to the maximum difference between any two connected grid point numbers in the band.

Examples of proper grid point sequencing for minimum bandwidth for one-dimensional systems are shown in figure 3.3. For open loops, a consecutive numbering system should be used as shown in figure 3.3a. Generally there is improvement in the accumulated roundoff error if the grid points are sequenced from the free end to the fixed end (where the temperature is prescribed).

For closed loops the grid points should be sequenced as shown in figure 3.3b. This model will have twice the semiband of the model shown in figure 3.3a. If the sequencing is as shown in figure 3.3c, the semiband will be half of that for the sequencing shown in figure 3.3b. However, the connection between grid points 1 and 8 will create a number of active columns equal to the semiband, and the net result is that the semiband of the first case is equal to the sum of the semiband and number of active columns for the second case. Since it takes about twice as long to process active columns as terms inside the band, the sequence shown in figure 3.3b is to be preferred.

Examples of grid point sequencing for two-dimensional surfaces are shown in figure 3.4. For planar or curved surfaces where the pattern of grid points tends to be rectangular, the sequencing shown in figure 3.4a will result in the shortest solution times. The semiband will be proportional to the number of grid points along the short direction of the pattern. If the pattern of grid points shown in figure 3.4a is made into a closed surface by connecting grid points 1 and 17, 2 and 18, etc., a number of active columns equal to the semiband will be created. An alternate sequencing for a closed loop is shown in figure 3.4b, where the semiband is proportional to twice the number of grid points in a row. For cylindrical or similar closed surfaces, the sequencing indicated in figure 3.4b is more efficient if the number of grid points in the circumferential direction is more than twice the number in the axial direction. If the number of grid points in the circumferential direction is less than twice the number in the axial direction, the sequencing indicated in figure 3.4a, with the consecutive numbering in the circumferential direction, is more efficient.

In general, sequences of grid points that generate active columns cannot be expected to shorten computing times substantially unless the semiband can be reduced by about two for each active column introduced. This is not likely to be the case for most surfaces. An exception is the case of radial patterns, where the sequencing indicated in figure 3.4c is the most efficient if there are more grid points on a circumferential line than on a radial line. In this case, the semiband is proportional to the number of grid points on a radial line, and the number of active columns is equal to the number of degrees of freedom at the center grid point. If there are more grid points on a radial line than a

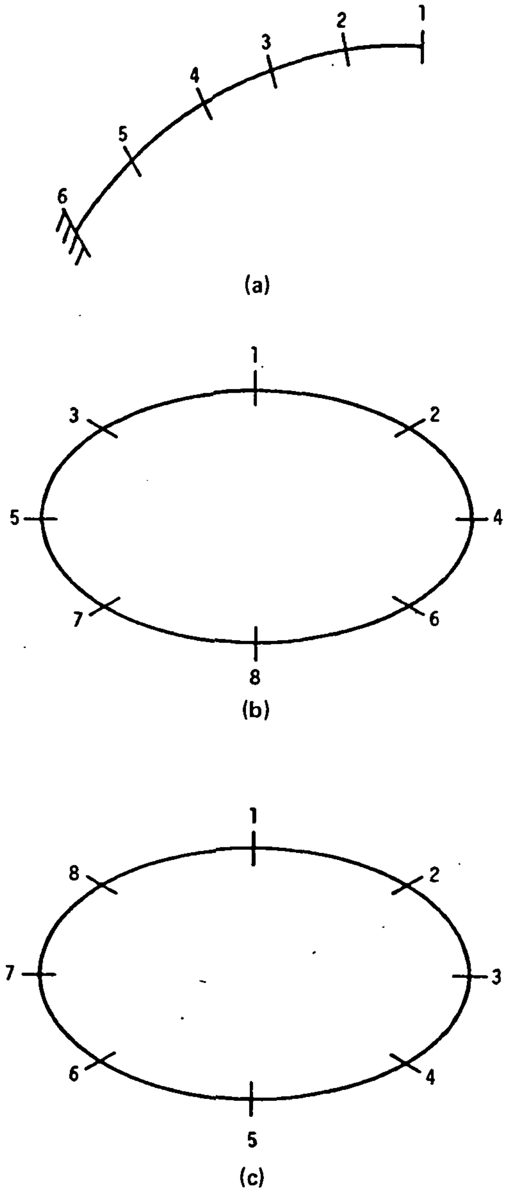


Figure 3.3. Grid point sequencing for one-dimensional systems.

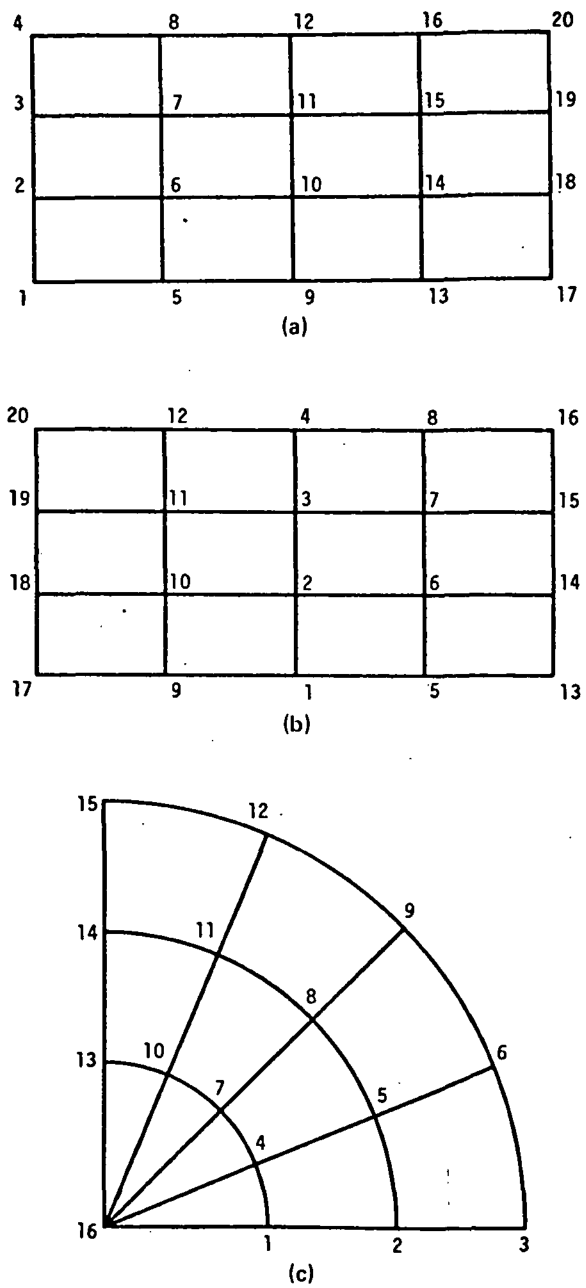


Figure 3.4. Grid point sequencing for two-dimensional surfaces.

circumferential line, the consecutive numbering should extend in the circumferential direction, beginning with the outermost circumferential ring. In this case, the semiband is proportional to the number of grid points on a circumferential line and there are no active columns.

If the grid points form a full circular pattern, the closure will create a number of active columns proportional to the number of grid points on a radial line if the grid points are numbered as shown in figure 3.4c. A more efficient scheme is to number the radial lines alternately, similar to the sequence shown for a rectangular array in figure 3.4b. This sequence will result in shorter solution times if the number of grid points on a circumferential line is greater than twice the number on a radial line. The central point must be sequenced at the end in order to limit the active columns to the number of degrees of freedom at the central point. If the central point is sequenced first, the number of active columns will be proportional to the number of radial lines. If the number of grid points on a circumferential line is less than twice the number of grid points on a radial line, the consecutive numbering should extend in the circumferential direction. This sequencing procedure will result in a semiband proportional to the number of grid points on a circumferential line and no active columns. If the central point does not exist, the sequencing problem is similar to that discussed for rectangular arrays in connection with figures 3.4a and 3.4b.

Sequencing problems for three-dimensional structures may be treated in two broad general classes. The first class consists of structural models that are compact, without appendages or connecting substructures. The second class consists of models that are composed of several substructures interconnected at a relatively small number of locations. Examples of the first type of model are solid structures, such as rectangular bars or cellular structures where an external shell is filled with bulkheads such as a submarine. For these types of structures the general procedure is to sequence the grid points in imaginary surfaces perpendicular to the largest axis of the structure. The grid point numbers are sequenced within each surface in the most effective way, beginning at one end of the structure and proceeding to the other end. Assuming that only adjacent surfaces are connected, the semiband will be proportional to the largest number of grid points in a surface.

Examples of the second type of model are airframes and radio telescopes. For these types of structures, the general procedure is to sequence the individual substructures in the most effective way and allow the degree of freedom associated with the connecting grid points to be treated as active columns. The computing time for terms outside the band is proportional to the total length of active columns, where the length of an active column is equal to the difference between the row number of the first nonzero term in the column and the row number of the extremity of the band. In sequencing the connecting grid points

for two substructures, the number of active columns is minimized by sequencing the connecting grid points after both substructures. However, in many cases, the connecting grid points can be advantageously sequenced between the two parts or among the points of the second part. This procedure tends to increase the number of active columns, but reduces the length of each one. Sequencing the connecting grid points first or among the points of the first part tends to maximize both the number of active columns and the lengths of each one.

Although scalar points are defined only in vector space, the pattern of their connections is used in a manner similar to that of geometric grid points for sequencing scalar points among themselves or with geometric grid points. Since scalar points introduced for dynamic analysis (extra points) are defined in connection with direct input matrices, the sequencing of these points is determined by direct reference to the positions of the added terms in the dynamic matrices.

The external identification numbers used for grid points may be selected in any manner the user desires. However, in order to preserve the bandwidth of the thermal conductance matrix, and hence to substantially reduce computing times when using the matrix inversion method, the internal sequencing of the grid points must not be arbitrary. In order to allow arbitrary external grid point numbers and still preserve sparsity in the triangular decomposition factors to the greatest extent possible, provision is made for the user to resequence the grid point numbers for internal operations. This feature also makes it possible to easily change the sequence if a poor initial choice is made. All output associated with grid points is identified with the external grid point numbers. The SEQGP card is used to resequence geometric grid points and scalar points. The SEQEP card is used to sequence the extra points in with the previously sequenced geometric grid points and scalar points.

c. Grid Point Properties – Some of the characteristics of the thermal model are introduced as properties of grid points, rather than as properties of heat conduction elements. Any of the various forms of direct matrix input are considered as describing the thermal model in terms of properties of grid points.

In transient thermal analysis, thermal capacitance and conductance properties may be provided, in part or entirely, as properties of grid points through the use of direct input matrices. The DMIG card is used to define direct input matrices for use in transient analysis. These matrices may be associated with geometric grid points, scalar points, or extra points introduced for transient analysis. The TF card is used to define transfer functions that are internally converted to direct matrix input.

2. HEAT CONDUCTION ELEMENTS

The heat conduction elements are a subset of the NASTRAN structural elements. These elements are summarized in the following table:

Heat Conduction Elements

Dimension	Type	Elements
1-D	Linear	BAR, RØD, CØNRØD, TUBE
2-D	Planar	TRMEM, TRIA1, TRIA2, QDMEM, QUAD1, QUAD2
	Solid of Revolution	TRIARG, TRAPRG
3-D	Solid	TETRA, WEDGE, HEXA1, HEXA2
-	Scalar	ELAS1, ELAS2, ELAS3, ELAS4

Heat conduction elements are defined on connection cards that identify the grid points to which the element is connected. The mnemonics for all such cards have a prefix of the letter "C," followed by an indication of the type of element, such as CBAR and CRØD. The order of the grid point identification defines the positive direction of the axis of a one-dimensional element and the positive surface of a plate element. The connection cards include additional orientation information when required. With a few exceptions, each connection card references a property definition card which can be referenced by many elements having the same properties. Thus, a large number of duplicate entries are eliminated.

A different class of elements that may also be used to model thermal conductance and thermal capacitance are the scalar elements which are connected between pairs of temperature unknowns (at either scalar or geometric grid points) or between a temperature unknown and a fixed zero (ground). Scalar spring elements are available to be used as thermal conductors, and scalar dampers are available to be used as thermal capacitors. These elements are useful for representing lumped properties of thermal conductance that cannot be conveniently modeled with the usual heat conduction elements. The elements CDAMPi (i=1,2,3,4) and CVISC are useful for representing lumped thermal capacitance between two selected unknown temperature variables or between one unknown temperature variable and a fixed zero (ground). It is possible, therefore, to construct a thermal model similar to a finite-difference based thermal network heat transfer computer model using only scalar elements and other appropriate cards for thermal boundary condition descriptions (see Sample Problem 19).[15] Sections 5.5 and 5.6 of the NASTRAN Theoretical Manual may be consulted for a detailed discussion of the use of scalar elements.

The property definition cards define geometric properties such as thicknesses and cross-sectional areas. The mnemonics for all such cards have a prefix of the letter "P," followed by some, or all of the characters used on the associated connection card, such as PBAR and PRØD. Except for the simplest elements, each property definition card will reference a material property card.

In some cases, the same finite element can be defined by using different bulk data cards. These alternate cards have been provided for convenience. For example, in the case of a rod element, the normal definition is accomplished with a connection card (CRØD) which references a property card (PRØD). However, an alternate definition uses a CØNRØD card which combines connection and property information on a single card. This is more convenient if a large number of rod elements all have different properties.

In the case of plate elements, a property card is provided for each type of element. In order to maintain uniformity in the relationship between connection cards and property cards, a number of connection card types contain the same information, such as the connection cards for the various types of triangular elements. Also, the property cards for triangular and quadrilateral elements of the same type contain the same information.

The trapezoidal solid of revolution element, TRAPRG, may be defined by a general quadrilateral ring (i.e., the top and bottom need not be perpendicular to the z-axis) for thermal applications. These thermal conduction elements are composed of constant gradient lines, triangles, and tetrahedra. The quadrilaterals are composed of overlapping triangles, and the wedges and hexahedra from subtetrahedra.

For the scalar elements, the most general definition of a thermal conductance (scalar spring) is given with a CELAS1 card. The associated properties are given on the PELAS card. The properties include the magnitude of the thermal conductance and a thermal capacitance. The CELAS2 defines a thermal conductance without reference to a property card. The CELAS3 card defines a thermal conductance that is connected only to scalar points and the properties are given on a PELAS card. The CELAS4 card defines a thermal conductance that is connected only to scalar points and without reference to a property card.

Scalar elements may be connected to ground without the use of constraint cards. Grounded connections are indicated on the connection card by leaving the appropriate scalar identification number blank. Since the values for scalar elements are not functions of material properties, no references to such cards are needed.

Regarding thermophysical properties, thermal conductivities, convective film coefficients and heat capacitances are given on the material property definition cards. The MAT4 card is used to define the properties for isotropic materials and the MAT5 card for anisotropic materials. Temperature-dependent conductivities and convective film coefficients are given on MATT4 and MATT5 Bulk Data cards, which

are limited to nonlinear steady-state analyses. The heat capacitance is the product of the density and the specific heat (ρC_p), and can be entered in the fourth and ninth fields of the MAT4 and MAT5 Bulk Data cards, respectively.

3. HEAT BOUNDARY ELEMENTS*

A heat boundary element (CHBDY) in conjunction with a PHBDY property card defines a surface area capable of accepting boundary heat fluxes and participating in radiative interchange. There are five basic types, called POINT, LINE, REV, AREA3, and AREA4. The extra special type, ELCYL, is for use only with QVECT directional radiant input. The HBDY element can contribute terms to the heat conductance and heat capacitance matrices. When a convective boundary condition coupling the boundary of a solid structure with a fluid of known temperature must be modeled, the thermal resistance across the boundary film and the associated thermal capacitance per unit area of the wetted contacting surface are specified on MAT4 Bulk Data cards. The known temperature of the fluid is specified with additional points, either GRID or SPØINT, which are referenced on the CHBDY continuation card.

The CHBDY Bulk Data cards are also used to define surfaces which participate in radiative heat exchanges. A list of CHBDY boundary elements must be specified by their element identification number on a RADLST Bulk Data card. Radiation exchange coefficients are specified on RADMTX Bulk Data cards. The radiation exchange coefficient is a product of the emitting surface area multiplied by the view factor between the emitting surface and the receiving surface in the diffuse-grey case. These coefficients are used in an internal SCRIPT-F routine which calculates the radiative interchange including all reflections.

The surface properties of emissivity and absorptivity are specified on the PHBDY Bulk Data cards.

4. CONSTRAINTS AND PARTITIONING

Constraints can be applied to describe desired boundary conditions, and provide other desired characteristics for the finite-element thermal model. There are two basic kinds of constraints:

Single-point constraints are used to specify the prescribed temperature at a point. The grid or scalar points are listed on SPC or SPC1 Bulk Data cards. The component on the data card can be "0" (scalar) or "1" (grid). This declares the specified temperatures to be in the u_s (constrained points) set. The method of specifying temperature varies in accordance with the problem type.

*The term "Boundary Surface Element" describing functions in the subsection 2.5.2 is, henceforth, replaced by the term "Heat BounDarY Element" (HBDY).

Algorithm	Value of u_s Used
Linear Steady-State	Values defined on selected SPC cards.
Nonlinear Steady-State	Values of the selected TEMP (MATERIAL) set. Use in conjunction with SPC1 cards.
Transient	$u_s = 0.0$ (special modeling techniques, such as a good conductor with a large power specified, can be used to enforce u(t)).

Multipoint constraints are linear relationships between temperatures at several grid points, and are specified on MPC cards. The first entry on the MPC card will be in the u_m set. The type of constraint is limited if nonlinear elements are present. If a member of set u_m touches a nonlinear element of the conductive or radiative type, the constraint relationship is restricted to be an "equivalence," which means that the value of the member of the u_m set must be set equal to one of the members of the u_n set (a point not multipoint constrained). Those points not touching nonlinear elements are not so limited. The user is responsible for satisfying this equivalence requirement by having only two entries on the MPC data card, with equal, but opposite in sign, coefficients.

The definitions of the u_m, u_n and u_s sets are given in section 3.3 of the NASTRAN Theoretical Manual. Finally, MPCADD and SPCADD cards may be used to combine different MPC and SPC sets, respectively.

5. THERMAL LOADS

Thermal loads may be internally generated heat or boundary heat fluxes. The latter includes prescribed heat fluxes, convective heat input, and radiative heat exchanges. The method of specifying thermal loads is different for steady-state and transient thermal analyses. With a few exceptions, the HBDY element must be used to define boundary surfaces in order to apply thermal loads to the conducting regions. The internally generated heat adding thermal energy into a heat conduction element is specified on a QVØL Bulk Data card which is one of the exceptions without reference to an HBDY element. The prescribed surface heat flux input can be specified for HBDY elements with the QBDY1 and QBDY2 Bulk Data cards. These two cards define a uniform heat flux and a spatially variable heat flux, respectively. A directional flux, such as solar radiation, impinges effective heat flux on the surface depending upon the angle between the flux vector and the surface normal of the

HBDY element. The directional flux is specified for HBDY elements with the QVECT Bulk Data card and the orientation of the HBDY element must be given (either implicitly or explicitly depending on the HBDY element type). Flux can also be specified directly without reference to an HBDY element with the QHBDY Bulk Data card.

Static thermal loads applied to steady-state thermal analyses are requested in Case Control with a LØAD card. All of the above load types plus SLØAD cards can be requested. Transient thermal loads are requested in Case Control with a DLØAD card, which selects time-dependent functions of TLØADi. Transient thermal loads may use DAREA,* QBDY1, QBDY2, QHBDY, QVECT, QVØL, and SLØAD Bulk Data cards. Although any number of load sets can be defined in the Bulk Data Deck, only those sets selected in the Case Control Deck, as described in section 3.4.2, will be used in the problem solution.

6. PARAMETER CARDS

The PARAMeter card is provided to specify the values of parameters used in DMAP sequences. A number of parameters in conjunction with the nonlinear steady-state and transient analyses may be entered via PARAM Bulk Data cards. For nonlinear steady-state analysis including radiative exchanges, the user can supply values on the PARAM Bulk Data cards for:

a. MAXIT – optional – the integer value of this parameter limits the maximum number of iterations (default = 4).

b. EPSHT – optional – the real value of this parameter is used to test the convergence of the solution (default = 0.001).

c. TABS – optional – the real value of this parameter is the absolute reference temperature (default = 0.0).

d. SIGMA – optional – the real value of this parameter is the Stefan-Boltzmann constant (default = 0.0).

e. HIRES – optional – a positive value of this parameter will cause the printing of the residual vectors following the execution of SSGHT for each iteration (default = –1).

For transient thermal analyses, the user can supply values on the PARAM Bulk Data cards for:

a. TABS – optional – the real value of this parameter is the absolute reference temperature (default = 0.0).

*This is the only load card which may not be used in all rigid formats–it is restricted to APP HEAT, SOL 9.

b. SIGMA – optional – the real value of this parameter is the Stefan-Boltzmann constant (default = 0.0).

c. BETA – optional – the real value of this parameter is used as a factor in the integration algorithm (default = 0.55).

d. RADLIN – optional – a positive integer value of this parameter causes some of the radiation effects to be linearized (default = -1).

It is to be noted that the parameter BETA, β, is used in both linear and nonlinear transient thermal analysis to affect the solution algorithms of the integration process. The detailed discussion concerning β is given in section 2.6.3. The other three parameters are used only in the nonlinear transient thermal analysis including radiative exchanges.

3.5.2 Format of Bulk Data Cards

The Bulk Data card format is variable to the extent that any quantity except the mnemonic can be punched anywhere within a specified 8- or 16-column field. The normal card uses an 8-column field as indicated in the following diagram:

Small Field Bulk Data Card

1a	2	3	4	5	6	7	8	9	10a
←8→	←8→	←8→	←8→	←8→	←8→	←8→	←8→	←8→	←8→

The mnemonic is punched in field 1 beginning in column 1. Fields 2-9 are for data items. The only limitations in data items are that they must lie completely within the designated field, have no imbedded blanks, and must be of the proper type, i.e., blank, integer, real, double precision, or BCD.* Real numbers may be encoded in various ways. Except in the case of zero or blank, a real number must contain a decimal point. For example, the real number 7.0 may be encoded as 7.0, .7E1, 0.7+1, 70.-1, 0.70+1, etc. A double precision number must contain both a decimal point and an exponent with the character D such as 7.0D0. Double precision data values are only allowed in a few situations, such as on the PARAM card. BCD data values consist of one to eight alphanumeric characters, the first of which must be alphabetic.

Normally field 10 is reserved for optional user identification. However, in the case of continuation cards field 10 (except column 73 which is not referenced) is used in conjunction with field 1 of the continuation card as an identifier and hence must contain a unique entry. The continuation card contains the symbol + in column 1 followed by the same seven characters that appeared in columns 74-80 of field 10 of the card that is being continued. This allows the data to be submitted as an unsorted deck.

*See SEQGP and SEQEP for exceptions.

The small field data card should be more than adequate for the kinds of data normally associated with thermal engineering problems. Since abbreviated forms of floating point numbers are allowed, up to seven significant decimal digits may be used in an eight-character field. Occasionally, however, the input is generated by another computer program or is available in a form where a wider field would be desirable. For this case, the larger field format with a 16-character data field is provided. Each logical card consists of two physical cards as indicated in the following diagram:

Large Field Bulk Data Card

1a	2	3	4	5	10a
←8→	←16→	←16→	←16→	←16→	←8→

1b	6	7	8	9	10b
←8→	←16→	←16→	←16→	←16→	←8→

The large field card is denoted by placing the symbol * after the mnemonic in field 1a and some unique character configuration in the last 7 columns of field 10a. The second physical card contains the symbol * in column 1 followed by the same seven characters that appeared after column 73 in field 10a of the first card. The second card may in turn be used to point to a large or small field continuation card, depending on whether the continuation card contains the symbol * or the symbol + in column 1. The use of multiple and large field cards are illustrated in the following examples:

Small Field Card with Small Field Continuation Card

TYPE									QED123
+ED123									

Large Field Card

TYPE*					QED124
*ED124					

Large Field Card with Large Field Continuation Card

TYPE*					QED301
*ED301					QED302
*ED302					QED305
*ED305					

Large Field Card Followed by a Small Field Continuation Card and a Large Field Continuation Card

<table>
<tr><td>TYPE*</td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td><td>QED462</td></tr>
<tr><td>*ED462</td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td><td>QED421</td></tr>
<tr><td>+ED421</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>QED361</td></tr>
<tr><td>*ED361</td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td><td>QED291</td></tr>
<tr><td>*ED291</td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td><td></td></tr>
</table>

Small Field Card with Large Field Continuation Card

<table>
<tr><td>TYPE</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>QED632</td></tr>
<tr><td>*ED632</td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td><td>QED204</td></tr>
<tr><td>*ED204</td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td><td></td></tr>
</table>

In the above examples column 73 arbitrarily contains the symbol Q in all cases where field 10 is used as a pointer. However, column 73 could have been left blank or the same symbol used in column 1 of the following card could have been used (i.e., the symbols * or +).

A sample NASTRAN data input sheet is shown in figure 3.5.

3.5.3 Bulk Data Card Descriptions

Detailed descriptions of selected Bulk Data cards that are frequently used in thermal analysis are presented in this section. Changes have been made to the names, descriptions and remarks on certain cards whose original terms and explanations were thermally meaningless, but the mnemonics of those cards have remained intact. Small field examples are given for each card along with a description of the contents of each field. In the Format and Example section of each card description, both a symbolic card format description and an example of an actual card are shown. Literal constants are shown in the card format section enclosed in quotes (e.g., "0"). Fields that are required to be blank are indicated in the card format section by whenever they are followed by nonblank fields or whenever such notation will clarify the card description.

The Input File Processor will produce error messages for any cards that do not have the proper format or which contain illegal data.

A list of Bulk Data cards commonly used in thermal modeling summarized and arranged in accordance with individual functional characteristics is as follows:

NASTRAN

PROJECT	NAME	
PROBLEM	DATE	PAGE OF

FIELD 1	FIELD 2	FIELD 3	FIELD 4	FIELD 5	FIELD 6	FIELD 7	FIELD 8	FIELD 9	FIELD 10
1 2 3 4 5 6 7 8	9 10 11 12 13 14 15 16	17 18 19 20 21 22 23 24	25 26 27 28 29 30 31 32	33 34 35 36 37 38 39 40	41 42 43 44 45 46 47 48	49 50 51 52 53 54 55 56	57 58 59 60 61 62 63 64	65 66 67 68 69 70 71 72	73 74 75 76 77 78 79 80
1 2 3 4 5 6 7 8	9 10 11 12 13 14 15 16	17 18 19 20 21 22 23 24	25 26 27 28 29 30 31 32	33 34 35 36 37 38 39 40	41 42 43 44 45 46 47 48	49 50 51 52 53 54 55 56	57 58 59 60 61 62 63 64	65 66 67 68 69 70 71 72	73 74 75 76 77 78 79 80

GSFC 6-61(5/70)

Figure 3.5. Sample NASTRAN data input sheet.

BULK DATA CARDS FREQUENTLY USED IN THERMAL ANALYSIS

Geometrical Definition:

CORDii
GRID, GRDSET, SEQGP
EPØINT
SPØINT

Element Connection:

Heat conduction:

CBAR, CRØD
CELAS2
CDAMP2
CQUAD2, CTRIA2
CHEXAi, CTETRA, CWEDGE (i = 1, 2, 3, 4)

Heat boundary:

CHBDY (POINT, LINE, REV, AREA3, AREA4, ELCYL)

Element Property Definition:

PBAR, PRØD
PQUAD2
PTRIA2
PHBDY

Thermophysical Property Definition:

MAT4, MAT5
MATT4, MATT5
TABLEMi (i = 1, 2, 3, 4)

Constraint Definition:

SPC, SPC1
MPC
ASET, ASET1, ØMIT, ØMIT1

Thermal Loading:

DAREA
DELAY
DLØAD
SLØAD
QBDY1, QBDY2

QHBDY
QVECT
QVØL
TABLEDi (i = 1, 2, 3, 4)
TLØADi (i = 1, 2)

Radiative Exchange Description:

RADLST
RADMTX

Misc:

DMI
NØLINi (i = 1, 2, 3, 4)
PARAM
TEMP, TEMPD
TF
TIC
TSTEP
/
$

Detailed descriptions of these selected Bulk Data cards that will follow are arranged in alphabetical order except the Comment ($) and Delete (/) cards which will be given first:

Input Data Card $ – Comment

Description: For user convenience in inserting commentary material into the unsorted echo of his input Bulk Data Deck. The $ card is otherwise ignored by the program. These cards will not appear in a sorted echo nor will they exist on the New Problem Tape.

Format and Example:

1	2	3	4	5	6	7	8	9	10
$	followed by any legitimate characters in card columns 2-80								
$	THIS IS A REMARK (*, '$$)–/								

Input Data Card / – Delete

Description: Delete cards are used to remove cards from either the Old Problem Tape on restart or the User's Master File.

Format and Example:

1	2	3	4	5	6	7	8	9	10
/	K1	K2							
/	4								

Field	Contents
K1	Sorted sequence number of first card in sequence to be removed
K2	Sorted sequence number of last card in sequence to be removed

Remarks:

1. The delete card causes Bulk Data cards having sort sequence numbers K1 through K2 to be removed from the Bulk Data Deck.
2. If K2 is blank, only card K1 is removed from the Bulk Data Deck.
3. If neither an Old Problem Tape nor a User's Master File are used in the current execution, the delete cards are ignored.

Input Data Card ASET – Selected Points

Description: Defines the grid and/or scalar points that the user desires to place in the analysis set.

Format and Example:

1	2	3	4	5	6	7	8	9	10
ASET	ID	C	ID	C	ID	C	ID	C	
ASET	16	1	23	1			1	0	

Field	Contents
ID	Grid or scalar point identification number (Integer > 0)
C	Component number, zero or blank for scalar points, 1 for grid points

Remarks:

1. Points specified on ASET cards may not be specified on ØMIT, ØMIT1, ASET1, SUPØRT, SPC or SPC1 cards nor may they appear as dependent points in multipoint constraint relations (MPC) or as permanent single-point constraints on a GRID card.
2. When ASET and/or ASET1 cards are present, all degrees of freedom not otherwise constrained will be placed in the Ø-set.

Input Data Card ASET1 – Selected Points

Description: Defines grid and/or scalar points that the user desires to place in the analysis set.

Format and Example:

1	2	3	4	5	6	7	8	9	10
ASET1	C	G	G	G	G	G	G	G	abc
ASET1	1	2	1	3	10	9	6	5	ABC

+bc	G	G	G	-etc.-					
+BC	7	8							

-etc.-

Alternate Form

ASET1	C	ID1	"THRU"	ID2					
ASET1	0	7	THRU	109					

Field	Contents
C	1 when point identification numbers are grid points; must be null or zero if point identification numbers are scalar points.
G,ID1, ID2	Grid or scalar point identification numbers (Integer > 0, ID1 < ID2)

Remarks:

1. A coordinate referenced on this card may not appear as a dependent coordinate in a multi-point constraint relation (MPC card), nor may it be referenced on an SPC, SPC1, ØMIT, ØMIT1, ASET, or SUPØRT card or on a GRID card as permanent single-point constraints.
2. When ASET and/or ASET1 cards are present, all degrees of freedom not otherwise constrained will be placed in the Ø-set.
3. If the alternate form is used, all of the grid (or scalar) points ID1 thru ID2 are assumed.

Input Data Card CBAR – Simple Beam Element Connection

Description: *Defines a 1-D heat conduction element (BAR) of the thermal model.*

Format and Example:

1	2	3	4	5	6	7	8	9	10
CBAR	EID	PID	GA	GB	X1, GO	X2	X3	F	abc
CBAR	2	39	7	3	13			2	123

+bc	PA	PB	Z1A	Z2A	Z3A	Z1B	Z2B	Z3B	
+23		513							

Field	Contents
EID	Unique element identification number (Integer > 0)
PID	Identification number of a PBAR property card (Integer > 0 or blank*)
GA, GB	Grid point identification numbers of connection points (Integer > 0; GA ≠ GB)
X1, X2, X3	Vector components measured in displacement coordinate system at GA to determine (with the vector from end A to end B) the orientation of the element coordinate system for the bar element (Real, $X1^2 + X2^2 + X3^2 > 0$ or blank*) (see below)
GO	Grid point identification number to optionally supply X1, X2, X3 (Integer > 0 or blank*) (see below)
F	Flag to specify the nature of fields 6–8 as follows:

	6	7	8
F = blank*			
F = 1	X1	X2	X3
F = 2	GO	blank/0	blank/0

[PA,PB]** Pin Flags for bar ends A and B (Up to 5 of the unique digits 1–6 anywhere in the field with no imbedded blanks; Integer > 0) (These degree of freedom codes refer to the element forces not to the global degrees of freedom). (See the Theoretical Manual section 5.2.)

*See the BARØR card for default options for fields 3, 6, 7, 8 and 9.

**Symbols in brackets denote those to be used only in the structural version of NASTRAN.

Field	Contents
[Z1A,Z2A,Z3A / Z1B,Z2B,Z3B]*	Offset vectors measured in displacement coordinate system at points GA and GB (Real)

Remarks:

1. Element identification numbers must be unique with respect to all other element identification numbers.

2. For an explanation of bar element geometry, see section 1.3.2 of the NASTRAN User's Manual.

3. Zero (0) must be used in fields 7 and 8 in order to override entries in these fields associated with F = 1 in field 9 on a BARØR card.

4. It is recommended that thermal analysts use the CRØD element if no structural compatibility is required.

*Symbols in brackets denote those to be used only in the structural version of NASTRAN.

Input Data Card CDAMP2 – Thermal Capacitance (Scalar Damper) Property and Connection

Description: Defines a thermal capacitance for a lumped thermal element without reference to a property value.

Format and Example:

1	2	3	4	5	6	7	8	9	10
CDAMP2	EID	B	G1	C1	G2	C2			
CDAMP2	16	-2.98	32	1					

Field	Contents
EID	Unique element identification number (Integer > 0)
B	The value of the thermal capacitance or scalar damper (Real)
G1,G2	Geometric grid point identification number (Integer ≥ 0)
C1,C2	Component number (1 or 0)

Remarks:

1. Scalar points may be used for G1 and/or G2 in which case the corresponding C1 and/or C2 must be zero or blank. Zero or blank may be used to indicate a grounded* terminal G1 or G2 with a corresponding blank or zero C1 or C2.
2. Element identification numbers must be unique with respect to all other element identification numbers.
3. This single card completely defines the element since no material or geometric properties are required.
4. The two connection points, (G1, C1) and (G2, C2), must be distinct.
5. For a discussion of the scalar elements, see section 5.6 of the NASTRAN Theoretical Manual.

*A grounded terminal is a scalar point or coordinate of a geometric grid point whose temperature is constrained to zero.

Input Data Card CELAS2 – Thermal Conductance (Scalar Spring) Property and Connection

Description: Defines a thermal conductance for a lumped thermal element without reference to a property value.

Format and Example:

1	2	3	4	5	6	7	8	9	10
CELAS2	EID	K	G1	C1	G2	C2	GE	S	
CELAS2	28	6.2+3	32	1	19	1			

Field	Contents
EID	Unique element identification number (Integer > 0)
K	The value of the thermal conductance (Real)
G1,G2	Geometric grid point identification number (Integer ⩾ 0)
C1,C2	Components number (1 or 0)
GE	Damping coefficient (Real)
[S] *	Stress coefficient (Real)

Remarks:

1. Scalar points may be used for G1 and/or G2 in which case the corresponding C1 and/or C2 must be zero or blank. Zero or blank may be used to indicate a grounded** terminal G1 or G2 with a corresponding blank or zero C1 or C2.
2. Element identification numbers must be unique with respect to all other element identification numbers.
3. This single card completely defines the element since no material or geometric properties are required.
4. The two connection points, (G1, C1) and (G2, C2), must be distinct.
5. For a discussion of the scalar elements, see section 5.6 of the NASTRAN Theoretical Manual.
6. In thermal runs (APP HEAT), the units for field 3 are Power/Degree (i.e., watts/°C).

*Symbols in brackets denote those to be used only in the structural version of NASTRAN.

**A grounded terminal is a scalar point or coordinate of a geometric grid point whose temperature is constrained to zero.

Input Data Card CHBDY – Heat Boundary Element

Description: Defines a boundary element for heat transfer analysis which is used for heat flux, thermal vector flux, convection and/or radiation.

Format and Example:

1	2	3	4	5	6	7	8	9	10
CHBDY	EID	PID	TYPE	G1	G2	G3	G4		
CHBDY	721	100	LINE	101	98				+BD721

+abc	GA1	GA2	GA3	GA4	V1	V2	V3		
+BD721	102	102			1.00	0.0	0.0		

Field	Contents
EID	Element identification number (Integer > 0)
PID	Property identification number (Integer > 0)
TYPE	Type of area involved. (Must be one of "PØINT", "LINE", "REV", "AREA3", "AREA4" or "ELCYL")
G1,G2, G3,G4	Grid point identification numbers of primary connected points (Integer > 0 or blank)
GA1,GA2, GA3,GA4	Grid or scalar point identification numbers of associated ambient points (Integer > 0 or blank)
V1,V2,V3	Vector (in the basic coordinate system) used for element orientation (real or blank)

Remarks:

1. The continuation card is not required.
2. The six types have the following characteristics:
   a. The "PØINT" type has one primary grid point, requires a property card, and the surface normal vector {V1, V2, V3} must be given if vectorial thermal flux is to be used.
   b. The "LINE" type has two primary grid points, requires a property card, and the surface normal vector is required if vectorial thermal flux is to be used.

c. The "REV" type has two primary grid points which must lie in the x–z plane of the basic coordinate system with $x > 0$. The defined area is a conical section with z as the axis of symmetry. A property card is required for convection, radiation, or vectorial thermal flux.

d. The "AREA3" and "AREA4" types have three and four primary grid points, respectively. These points define a triangular or quadrilateral surface and must be ordered to go around the boundary. A property card is required for convection, radiation, or vectorial thermal flux.

e. The "ELCYL" type (elliptic cylinder) has two connected primary grid points, it requires a property card, and if vectorial thermal flux is used, the vector must be nonzero.

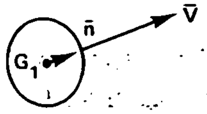
3. A property card, PHBDY, is used to define the associated area factors, the emissivity, the absorptivity, and the principal radii of the elliptic cylinder. The material coefficients used for convection and thermal capacity are referenced by the PHBDY card. See this card description for details.

4. The associated points, GA1, GA2, etc., may be either grid or scalar points, and are used to define the ambient temperature for a convection field. These points correspond to the primary points G1, G2, etc., and the number of them depends on the TYPE option, but they need not be unique. Their values may be set in statics with an SPC card, or they may be connected to other elements. If any field is blank, the ambient temperature associated with that grid point is assumed to be zero.

5. Heat flux may be applied to this element with QBDY1 or QBDY2 cards.

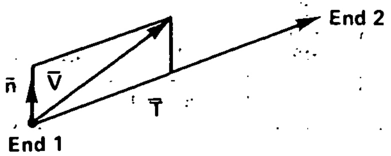
6. Vectorial thermal flux from a directional source may be applied to this element with a QVECT card. See figure 3:6 for the definition of the normal vector for each element type.

Type = PØINT



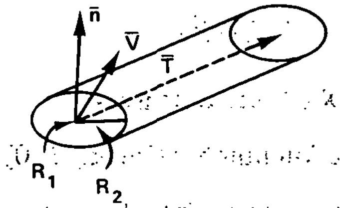
The unit normal vector is given by $\overline{n} = \overline{V}/|\overline{V}|$, where $\overline{V}$ is given in the basic coordinate system (see CHBDY data card, fields 16–18).

Type = LINE



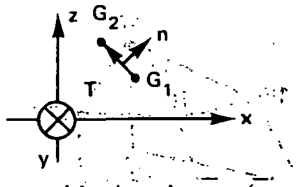
The unit normal lies in the plane of $\overline{V}$ and $\overline{T}$, is perpendicular to $\overline{T}$, and is given by $\overline{n} = (\overline{T} \times (\overline{V} \times \overline{T}))/|\overline{T} \times (\overline{V} \times \overline{T})|$.

Type = ELCYL



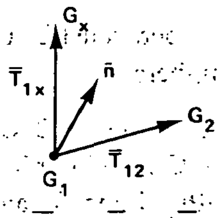
The same logic is used to determine $\overline{n}$ as for type = LINE. The "radius" R_1 is in the $\overline{n}$ direction, and R_2 is perpendicular to $\overline{n}$ and $\overline{T}$ (see fields 7 and 8 of PHBDY card).

Type = REV



The unit normal lies in the x-z plane, and is given by $\overline{n} = (\overline{e}_y \times \overline{T})/|\overline{e}_y \times \overline{T}|$. $\overline{e}_y$ is the unit vector in the y direction.

Type = AREA3 or AREA4



The unit normal vector is given by $\overline{n} = (\overline{T}_{12} \times \overline{T}_{1x})/|\overline{T}_{12} \times \overline{T}_{1x}|$, where x = 3 for triangles and x = 4 for quadrilaterals.

Figure 3.6. HBDY element orientation (for QVECT flux).

Input Data Card CHEXAi – Hexahedron Element Connection

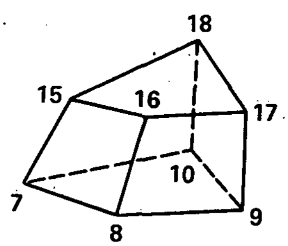
Description: Defines two types of hexahedron elements (3-dimensional solid with 8 vertices and 6 quadrilateral faces, HEXAi) of the thermal model.

Format and Example:

1	2	3	4	5	6	7	8	9	10
CHEXAi	EID	MID	G1	G2	G3	G4	G5	G6	abc
CHEXA2	15	2	7	8	9	10	15	16	ABC

+bc	G7	G8							
+BC	17	18							

Field	Contents
CHEXAi	CHEXA1 or CHEXA2 (see Remark 4)
EID	Element identification number (Integer > 0)
MID	Material identification number (Integer > 0)
G1,. . .,G8	Grid point identification numbers of connection points (Integers > 0, $G1 \neq G2 \neq \ldots \neq G8$)



Remarks:

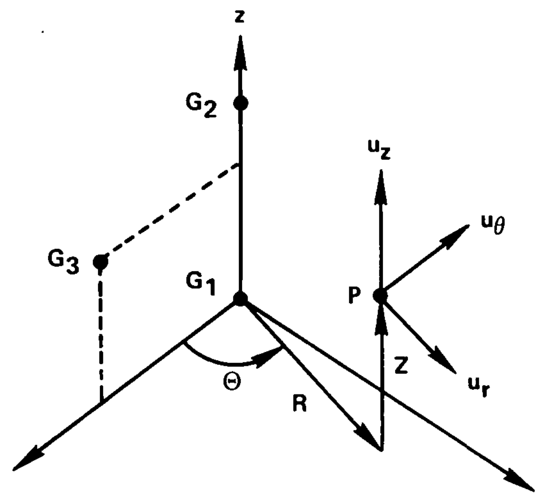
1. Element identification numbers must be unique with respect to all other element identification numbers.
2. The order at the grid points is: G1, G2, G3, G4 in order around one quadrilateral face. G5, G6, G7, G8 are in order in the same direction around the opposite quadrilateral, with G1 and G5 along the same edge.
3. The quadrilateral faces must be nearly planar.

4. CHEXA1 represents the element as 5 tetrahedra, CHEXA2 represents the element as 10 overlapping tetrahedra.

5. Thermophysical properties may be defined with either a MAT4 or MAT5 card.

Input Data Card CØRD1C – Cylindrical Coordinate System Definition

Description: Defines a cylindrical coordinate system by reference to three grid points. These points must be defined in coordinate systems whose definition does not involve the coordinate system being defined. The first point is the origin, the second lies on the z-axis, and the third lies in the plane of the azimuthal origin.



Format and Example:

1	2	3	4	5	6	7	8	9	10
CØRD1C	CID	G1	G2	G3	CID	G1	G2	G3	
CØRD1C	3	16	32	19					

Field	Contents
CID	Coordinate system identification number (Integer > 0)
G1, G2, G3	Grid point identification numbers (Integer > 0; G1 ≠ G2 ≠ G3)

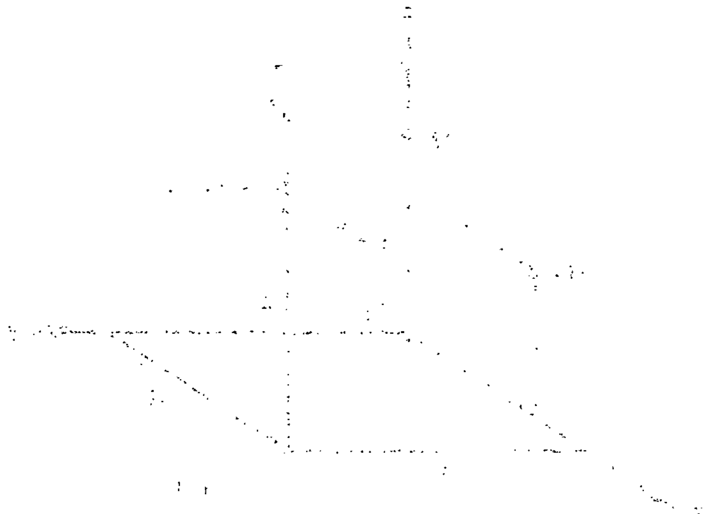
Remarks:

1. Coordinate system identification numbers on all CØRD1R, CØRD1C, CØRD1S, CØRD2R, CØRD2C, and CØRD2S cards must all be unique.
2. The three points G1, G2, G3 must be noncolinear.
3. The location of a grid point (P in the sketch) in this coordinate system is given by (R, Θ, Z) where Θ is measured in degrees.

4. The displacement coordinate directions at P are dependent on the location of P as shown above by (u_r, u_θ, u_z).

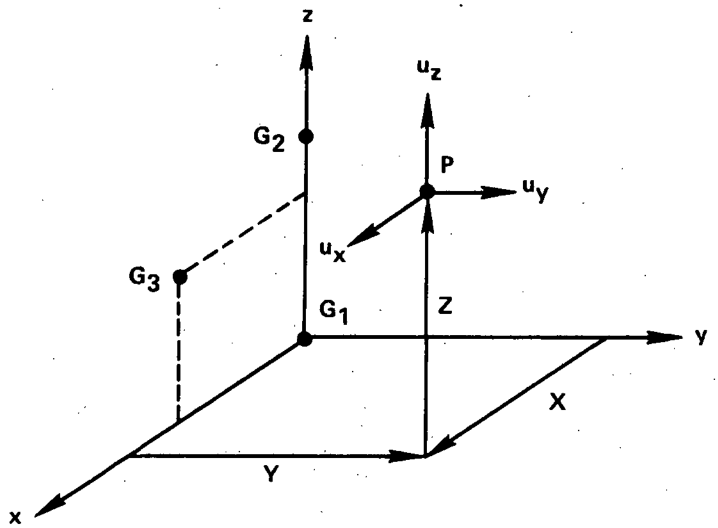
5. Points on the z-axis may not have their displacement directions defined in this coordinate system since an ambiguity results.

6. One or two coordinate systems may be defined on a single card.



Input Data Card CØRD1R – Rectangular Coordinate System Definition

Description: Defines a rectangular coordinate system by reference to three grid points. These points must be defined in coordinate systems whose definition does not involve the coordinate system being defined. The first point is the origin, the second lies on the z-axis, and the third lies in the x-z plane.



Format and Example:

1	2	3	4	5	6	7	8	9	10
CØRD1R	CID	G1	G2	G3	CID	G1	G2	G3	
CØRD1R	3	16	32	19					

Field	Contents
CID	Coordinate system identification number (Integer > 0)
G1, G2, G3	Grid point identification numbers (Integer > 0; G1 ≠ G2 ≠ G3)

Remarks:

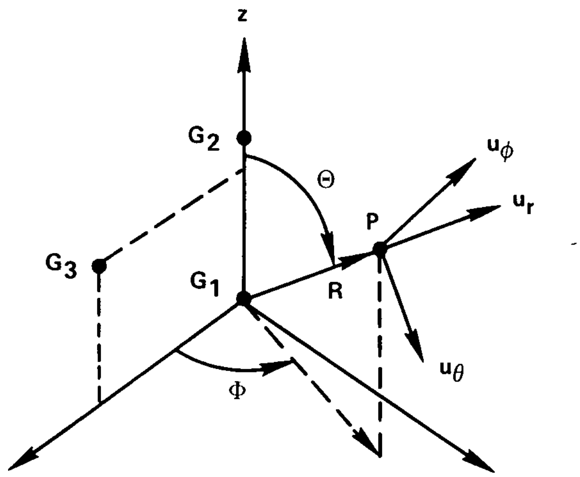
1. Coordinate system identification numbers on all CØRD1R, CØRD1C, CØRD1S, CØRD2R, CØRD2C, and CØRD2S cards must all be unique.
2. The three points G1, G2, G3 must be noncolinear.
3. The location of a grid point (P in the sketch) in this coordinate system is given by (X, Y, Z).

4. The displacement coordinate directions at P are shown above by (u_x, u_y, u_z).

5. One or two coordinate systems may be defined on a single card.

Input Data Card CØRD1S – Spherical Coordinate System Definition

Description: Defines a spherical coordinate system by reference to three grid points. These points must be defined in coordinate systems whose definition does not involve the coordinate system being defined. The first point is the origin, the second lies on the z-axis, and the third lies in the plane of the azimuthal origin.



Format and Example:

1	2	3	4	5	6	7	8	9	10
CØRD1S	CID	G1	G2	G3	CID	G1	G2	G3	
CØRD1S	3	16	32	19					

Field	Contents
CID	Coordinate system identification number (Integer > 0)
G1, G2, G3	Grid point identification numbers (Integer > 0; G1 ≠ G2 ≠ G3)

Remarks:

1. Coordinate system identification numbers on all CØRD1R, CØRD1C, CØRD1S, CØRD2R, CØRD2C, and CØRD2S cards must all be unique.
2. The three points G1, G2, G3 must be noncolinear.

3. The location of a grid point (P in the sketch) in this coordinate system is given by (R, Θ, Φ) where Θ and Φ are measured in degrees.

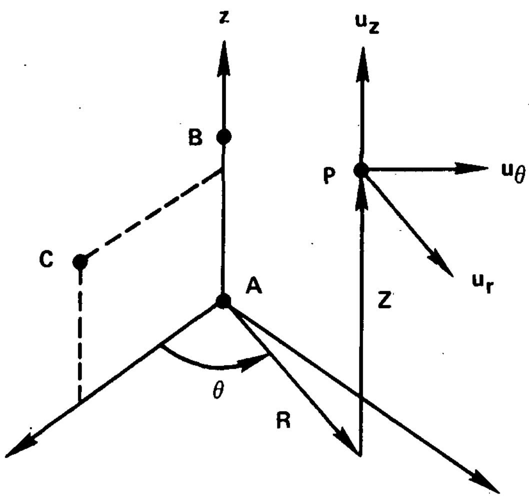
4. The displacement coordinate directions at P are dependent on the location of P as shown above by (u_r, u_θ, u_ϕ).

5. Points on the polar axis may not have their displacement directions defined in this coordinate system since an ambiguity results.

6. One or two coordinate systems may be defined on a single card.

Input Data Card CØRD2C – Cylindrical Coordinate System Definition

Description: Defines a cylindrical coordinate system by reference to the coordinates of three points. The first point defines the origin; the second point defines the direction of the z-axis; the third lies in the plane of the azimuthal origin. The reference coordinate must be independently defined.



Format and Example:

1	2	3	4	5	6	7	8	9	10
CØRD2C	CID	RID	A1	A2	A3	B1	B2	B3	ABC
CØRD2C	3	17	-2.9	1.0	0.0	3.6	0.0	1.0	123
+BC	C1	C2	C3						
+23	5.2	1.0	-2.9						

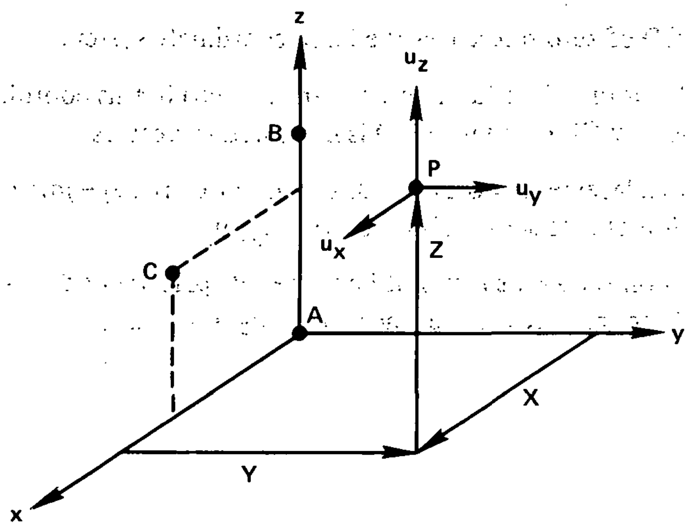
Field	Contents
CID	Coordinate system identification number (Integer > 0)
RID	Reference to a coordinate system which is defined independently of new coordinate system (Integer ⩾ 0 or blank)
A1, A2, A3 B1, B2, B3 C1, C2, C3	Coordinates of three points in coordinate system defined in field 3 (Real)

Remarks:

1. Continuation card must be present.
2. The three points (A1, A2, A3), (B1, B2, B3), (C1, C2, C3) must be unique and noncolinear. Noncolinearity is checked by the geometry processor.
3. Coordinate system identification numbers on all CØRD1R, CØRD1C, CØRD1S, CØRD2R, CØRD2C, and CØRD2S cards must all be unique.
4. An RID of zero references the basic coordinate system.
5. The location of a grid point (P in the sketch) in this coordinate system is given by (R, Θ, Z) where Θ is measured in degrees.
6. The displacement coordinate directions at P are dependent on the location of P as shown above by (u_r, u_θ, u_z).
7. Points on the z-axis may not have their displacement direction defined in this coordinate system since an ambiguity results.

Input Data Card C∅RD2R – Rectangular Coordinate System Definition

Description: Defines a rectangular coordinate system by reference to the coordinates of three points. The first point defines the origin; the second point defines the direction of the z-axis; the third point defines a vector which, with the z-axis, defines the x-z plane. The reference coordinate must be independently defined.



Format and Example:

1	2	3	4	5	6	7	8	9	10
C∅RD2R	CID	RID	A1	A2	A3	B1	B2	B3	ABC
C∅RD2R	3	17	-2.9	1.0	0.0	3.6	0.0	1.0	123
+BC	C1	C2	C3						
+23	5.2	1.0	-2.9						

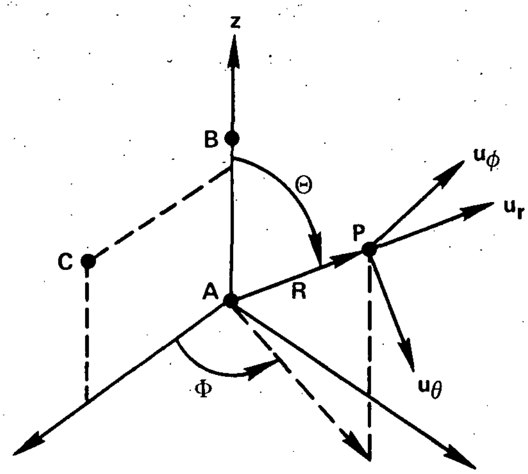
Field	Contents
CID	Coordinate system identification number (Integer > 0)
RID	Reference to a coordinate system which is defined independently of new coordinate system (Integer ⩾ 0 or blank)
A1, A2, A3 B1, B2, B3 C1, C2, C3	Coordinates of three points in coordinate system defined in field 3 (Real)

Remarks:

1. Continuation card <u>must</u> be present.
2. The three points (A1, A2, A3), (B1, B2, B3), (C1, C2, C3) must be unique and noncolinear. Noncolinearity is checked by the geometry processor.
3. Coordinate system identification numbers on all CØRD1R, CØRD1C, CØRD1S, CØRD2R, CØRD2C, and CØRD2S cards must all be unique.
4. An RID of zero references the basic coordinate system.
5. The location of a grid point (P in the sketch) in this coordinate system is given by (X, Y, Z).
6. The displacement coordinate directions at P are shown by (u_x, u_y, u_z).

Input Data Card CØRD2S – Spherical Coordinate System Definition

Description: Defines a spherical coordinate system by reference to the coordinates of three points. The first point defines the origin; the second point defines the direction of the z-axis; the third lies in the plane of the azimuthal origin. The reference coordinate must be independently defined.



Format and Example:

1	2	3	4	5	6	7	8	9	10
CØRD2S	CID	RID	A1	A2	A3	B1	B2	B3	ABC
CØRD2S	3	17	-2.9	1.0	0.0	3.6	0.0	1.0	123
+BC	C1	C2	C3						
+23	5.2	1.0	-2.9						

Field	Contents
CID	Coordinate system identification number (Integer > 0)
RID	Reference to a coordinate system which is defined independently of new coordinate system (Integer ⩾ 0 or blank)
A1, A2, A3 B1, B2, B3 C1, C2, C3	Coordinate of three points in coordinate system defined in field 3 (Real)

Remarks:

1. Continuation card must be present.
2. The three points (A1, A2, A3), (B1, B2, B3), (C1, C2, C3) must be unique and noncolinear. Noncolinearity is checked by the geometry processor.
3. Coordinate system identification numbers on all CØRD1R, CØRD1C, CØRD1S, CØRD2R, CØRD2C, and CØRD2S must all be unique.
4. An RID of zero references the basic coordinate system.
5. The location of a grid point (P in the sketch) in this coordinate system is given by (R, Θ, Φ) where Θ and Φ are measured in degrees.
6. The displacement coordinate directions at P are shown above by (u_r, u_θ, u_ϕ).
7. Points on the polar axis may not have their displacement directions defined in this coordinate system since an ambiguity results.

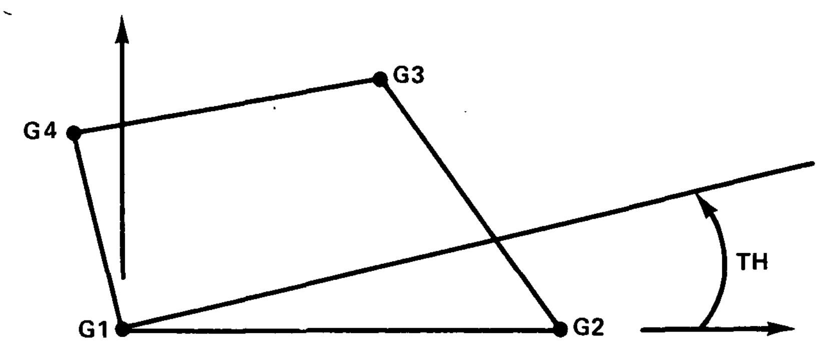
Input Data Card CQUAD2 – Quadrilateral Element Connection

Description: Defines a homogeneous quadrilateral heat conduction element (QUAD2) of the thermal model.

Format and Example:

1	2	3	4	5	6	7	8	9	10
CQUAD2	EID	PID	G1	G2	G3	G4	TH		
CQUAD2	72	13	13	14	15	16	29.2		

Field	Contents
EID	Element identification number (Integer > 0)
PID	Identification number of a PQUAD2 property card (Default is EID) (Integer > 0)
G1,G2, G3,G4	Grid point identification numbers of connection points (Integer > 0; G1 ≠ G2 ≠ G3 ≠ G4)
TH	Material property orientation angle in degrees (Real). The sketch below gives the sign convention for TH.



Remarks:
1. Element identification numbers must be unique with respect to all other element identification numbers.
2. Grid points G1 through G4 must be ordered consecutively around the perimeter of the element.
3. All interior angles must be less than 180°.

Input Data Card CRØD – Rod Element Connection

Description: Defines a 1-D heat conduction element (RØD) of the thermal model.

Format and Example:

1	2	3	4	5	6	7	8	9	10
CRØD	EID	PID	G1	G2	EID	PID	G1	G2	
CRØD	12	13	21	23	3	12	24	5	

Field	Contents
EID	Element identification number (Integer > 0)
PID	Identification number of a PRØD property card (Default is EID) (Integer > 0)
G1,G2	Grid point identification numbers of connection points (Integar > 0; G1 ≠ G2)

Remarks:

1. Element identification numbers must be unique with respect to all other element identification numbers.
2. See CØNRØD for alternative method of rod definition.
3. One or two RØD elements may be defined on a single card.

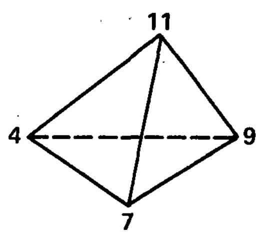
Input Data Card CTETRA – Tetrahedron Element Connection

Description: Defines a tetrahedron element (3-dimensional solid with 4 vertices and 4 triangular faces, TETRA) of the thermal model.

Format and Example:

1	2	3	4	5	6	7	8	9	10
CTETRA	EID	MID	G1	G2	G3	G4			
CTETRA	15	2	4	7	9	11			

Field	Contents
EID	Element identification number (Integer > 0)
MID	Material identification number (Integer > 0)
G1,G2, G3,G4	Grid point identification numbers of connection points (Integers > 0, G1 ≠ G2 ≠ G3 ≠ G4)



Remarks:

1. Element identification numbers must be unique with respect to all other element identification numbers.
2. Thermophysical properties may be defined with either a MAT4 or MAT5 card.

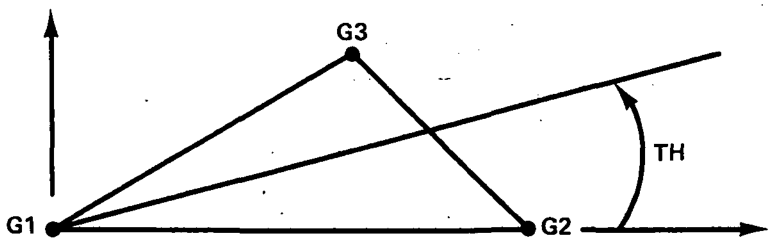
Input Data Card CTRIA2 – Triangular Element Connection

Description: Defines a 2-D triangular heat conduction element (TRIA2) of the thermal model.

Format and Example:

1	2	3	4	5	6	7	8	9	10
CTRIA2	EID	PID	G1	G2	G3	TH			
CTRIA2	16	2	12	1	3	16.2			

Field	Contents
EID	Element identification number (Integer > 0)
PID	Identification number of a PTRIA2 property card (Integer > 0)
G1,G2,G3	Grid point identification numbers of connection points (Integer > 0; $G1 \neq G2 \neq G3$)
TH	Material property orientation angle in degrees (Real) – The sketch below gives the sign convention for TH.



Remarks:

1. Element identification numbers must be unique with respect to all other element identification numbers.
2. Interior angles must be less than 180°.

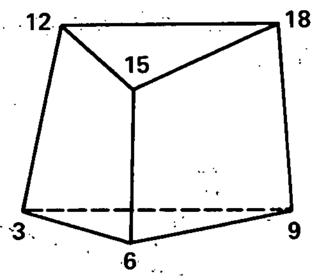
Input Data Card CWEDGE – Wedge Element Connection

Description: Defines a wedge element (3-dimensional solid, with three quadrilateral faces and two opposing triangular faces, WEDGE) of the thermal model.

Format and Example:

1	2	3	4	5	6	7	8	9	10
CWEDGE	EID	MID	G1	G2	G3	G4	G5	G6	
CWEDGE	15	2	3	6	9	12	15	18	

Field	Contents
EID	Element identification number (Integer > 0)
MID	Material identification number (Integer > 0)
G1,...,G6	Grid point identification numbers of connection points (Integers > 0, $G1 \neq G2 \neq \ldots \neq G6$)



Remarks:

1. Element identification numbers must be unique with respect to all other element identification numbers.
2. The order of the grid points is: G1, G2, G3 on one triangular face, G4, G5, G6 at the other triangular face. G1, G4 on a common edge, G2, G5 on a common edge.
3. The quadrilateral faces must be nearly planar.
4. Thermophysical properties may be defined with either a MAT4 or MAT5 card.

Input Data Card DAREA – Dynamic Load Scale Factor

Description: This card is used in conjunction with the TLØAD1 and TLØAD2 data cards and defines the point where the dynamic load is to be applied with the scale factor A.

Format and Example:

1	2	3	4	5	6	7	8	9	10
DAREA	SID	P	C	A	P	C	A		
DAREA	3	6	1	8.2	15	1	10.1		

Field	Contents
SID	Identification number of DAREA set (Integer > 0)
P	Grid or scalar point identification number (Integer > 0)
C	Component number (1 for grid point; blank or 0 for scalar point)
A	Scale factor A for the designated coordinate (Real)

Remarks:

1. One or two dynamic load time delays may be defined on a single card.
2. Several versions of the $\overline{\text{N}}$TA require that a DAREA card be supplied for each TLØADi card which is input which has a unique value in field 3.

Input Data Card DELAY – Dynamic Load Time Delay

Description: This card is used in conjunction with the TLØAD1 and TLØAD2 data cards and defines the time delay term τ in the equations of the loading function.

Format and Example:

1	2	3	4	5	6	7	8	9	10
DELAY	SID	P	C	T	P	C	T		
DELAY	5	21	1	4.25	7	0	8.1		

Field	Contents
SID	Identification number of DELAY set (Integer > 0)
P	Grid or scalar point identification number (Integer > 0)
C	Component number (1 for grid point, blank or 0 for scalar point)
T	Time delay τ for designated coordinate (Real)

Remarks: One or two dynamic load time delays may be defined on a single card.

Input Data Card DLØAD – Dynamic Load Combination (Superposition)

Description: Defines a dynamic loading condition for transient problems as a linear combination of load sets defined via TLØAD1 or TLØAD2 cards.

Format and Example:

1	2	3	4	5	6	7	8	9	10
DLØAD	SID	S	S1	L1	S2	L2	S3	L3	+abc
DLØAD	17	1.0	2.0	6	-2.0	7	2.0	8	+A

+abc	S4	L4		-etc.-					
+A	-2.0	9							

-etc.-

Field	Contents
SID	Load set identification number (Integer > 0)
S	Scale Factor (Real)
Si	Scale Factors (Real)
Li	Load set identification numbers defined via card types enumerated above (Integer > 0)

Remarks:

1. The load vector being defined by this card is given by

$$\{P\} = S\sum_i S_i\{P_i\}.$$

2. The Li must be unique.
3. SID must be unique from all Li.
4. Nonlinear transient loads may not be included; they are selected separately in the Case Control Deck.
5. Linear load sets must be selected in the Case Control Deck (DLØAD=SID) to be used by NASTRAN.
6. A DLØAD card may not reference a set identification number defined by another DLØAD card.
7. TLØAD1 and TLØAD2 loads may be combined only through the use of the DLØAD card.
8. SID must be unique for all TLØAD1 and TLØAD2 cards.

Input Data Card DMI – Direct Matrix Input

Description: Used to define matrix data blocks directly. Generates a matrix of the form

$$[A] = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2n} \\ \vdots & \vdots & & \vdots \\ A_{m1} & \cdots & \cdots & A_{mn} \end{bmatrix}$$

where the elements A_{ij} may be real or complex single-precision numbers.

Formats and Example: (The first logical card is a header card.)

1	2	3	4	5	6	7	8	9	10
DMI	NAME	"0"	FØRM	TIN	TØUT		M	N	
DMI	QQQ	0	2	3	3		4	2	

DMI	NAME	J	I1	A(I1,J)	A(I1+1,J)		etc.	I2	+abc
DMI	QQQ	1	1	1.0	2.0	3.0	4.0	3	+1
+abc	A(I2,J)		etc.						
+1	5.0	6.0							

DMI	QQQ	2	2	6.0	7.0	4	8.0	9.0	

(etc. for each nonnull column)

Field	Contents
NAME	Any NASTRAN BCD value (1–8 alphanumeric characters, the first of which must be alphabetic) which will be used in the DMAP sequence to reference the data block
FØRM	1 Square matrix (not symmetric) 2 General rectangular matrix 6 Symmetric matrix
TIN	Type of matrix being input as follows: 1 Real, single-precision (One field is used per element) 3 Complex, single-precision (Two fields are used per element)

Field	Contents
TØUT	Type of matrix which will be created 1 Real, single-precision 3 Complex, single-precision 2 Real, double-precision 4 Complex, double-precision
M	Number of rows in A (Integer > 0)
N	Number of columns in A (Integer > 0)
J	Column number of A (Integer > 0)
I1,I2,etc.	Row number of A (Integer > 0)
A(Ix,J)	Element of A (See TIN) (Real)

Remarks:

1. The user must write a DMAP (or make alterations to a rigid format) in order to use the DMI feature since he is defining a data block. All of the rules governing the use of data blocks in DMAP sequences apply. In the example shown above, the data block QQQ is defined to be the complex, single-precision rectangular 4x2 matrix

$$[QQQ] = \begin{bmatrix} (1.0, 2.0) & (0.0, 0.0) \\ (3.0, 4.0) & (6.0, 7.0) \\ (5.0, 6.0) & (0.0, 0.0) \\ (0.0, 0.0) & (8.0, 9.0) \end{bmatrix}$$

   The DMAP data block NAME (QQQ in the example) will appear in the initial FIAT and the data block will initially appear on the Data Pool File (PØØL).

2. A limit to the number of DMI's which may be defined is set by the size of the Data Pool Dictionary. The total number of DMI's may not exceed this size.

3. There are a number of reserved words which may not be used for DMI names. Among these are PØØL, NPTP, ØPTP, UMF, NUMF, PLT1, PLT2, INPT, GEØM1, GEØM2, GEØM3, GEØM4, GEØM5, EDT, MPT, EPT, DIT, DYNAMICS, IFPFILE, AXIC, FØRCE, MATPØØL, PCDB, XYCDB, CASECC, any DTI names, and SCRATCH1 through SCRATCH9.

4. Field 3 of the header card must contain an integer 0.

5. For symmetric matrices, the entire matrix must be input.

6. Only nonzero terms need be entered.

Input Data Card EPØINT – Extra Point

Description: Defines extra points of the thermal model for use in transient problems.

Format and Example:

1	2	3	4	5	6	7	8	9	10
EPØINT	ID	ID	ID	ID	ID	ID	ID	ID	
EPØINT	3	18	1	4	16	2			

Alternate Form

EPØINT	ID1	"THRU"	ID2						
EPØINT	17	THRU	43						

Field	Contents
ID,ID1,ID2	Extra point identification number (Integer > 0; ID1 < ID2)

Remarks:

1. Identification numbers of points defined by GRID, SPØINT and EPØINT cards and scalar points defined on scalar element connection cards must all be unique.
2. This card is used to define coordinates used in transfer function definitions (see TF card).
3. If the alternate form is used, extra points ID1 through ID2 are defined.

Input Data Card GRDSET – Grid Point Default

Description: Defines default options for fields 3, 7 and 8 of all GRID cards.

Format and Example:

1	2	3	4	5	6	7	8	9	10
GRDSET		CP				CD	PS		
GRDSET		16				32	3456		

Field	Contents
CP	Identification number of coordinate system in which the location of the grid point is defined (Integer ≥ 0)
[CD]*	Identification number of coordinate system in which displacements are measured at grid point (Integer ≥ 0)
PS	Permanent single-point constraint associated with grid point (a 1 indicates a permanent single-point constraint) (Integer ≥ 0)

Remarks:

1. The contents of fields 3, 7 or 8 of this card are assumed for the corresponding fields of any GRID card whose field 3, 7 and 8 are blank. If any of these fields on the GRID card are blank, the default option defined by this card occurs for that field. If no permanent single-point constraints are desired or one of the coordinate systems is basic, the default may be overridden on the GRID card by making one of fields 3, 7 or 8 zero (rather than blank). Only one GRDSET card may appear in the user's Bulk Data Deck.
2. The primary purpose of this card is to minimize the burden of preparing data for problems with a large amount of repetition.
3. At least one of the entries CP, CD or PS must be nonzero.

*Symbols in brackets denote those to be used only in the structural version of NASTRAN.

Input Data Card GRID – Grid Point

Description: Defines the location of a geometric grid point of the thermal model, and its permanent single-point constraint.

Format and Example:

1	2	3	4	5	6	7	8	9	10
GRID	ID	CP	X1	X2	X3	CD	PS		
GRID	2	3	1.0	2.0	3.0		316		

Field	Contents
ID	Grid point identification number (Integer > 0)
CP	Identification number of coordinate system in which the location of the grid point is defined (Integer ≥ 0 or blank*)
X1,X2,X3	Location of the grid point in coordinate system CP (Real)
[CD]**	Identification number of coordinate system in which displacements, degrees of freedom, constraints, and solution vectors are defined at the grid point (Integer ≥ 0 or blank*)
PS	Permanent single-point constraint associated with grid point (a 1 indicates a permanent single-point constraint) (Integer ≥ 0 or blank*)

Remarks:

1. All grid point identification numbers must be unique with respect to all other structural, scalar, and fluid points.

2. The meaning of X1, X2 and X3 depend on the type of coordinate system, CP, as follows: (see CØRD card descriptions)

Type	X1	X2	X3
Rectangular	X	Y	Z
Cylindrical	R	θ(degrees)	Z
Spherical	R	θ(degrees)	ϕ(degrees)

3. The collection of all CD coordinate systems defined on all GRID cards is called the Global Coordinate System. All degrees-of-freedom, constraints, and solution vectors are expressed in the Global Coordinate System.

*See the GRDSET card for default options for fields 3, 7 and 8.

**Symbols in brackets denote those to be used only in the structural version of NASTRAN.

Input Data Card MAT4 – Thermophysical Property Definition

Description: Defines the thermophysical properties for temperature-independent, isotropic materials.

Format and Example:

1	2	3	4	5	6	7	8	9	10
MAT4	MID	K	CP						
MAT4	103	0.6	0.2						

Field	Contents
MID	Material identification number (Integer > 0)
K	Thermal conductivity (Real > 0.0), or convective film coefficient
CP	Thermal capacity per unit volume (Real > 0.0 or blank), or thermal capacity of a film layer per unit area

Remarks:

1. The material identification number may be the same as a MAT1, MAT2, or MAT3 card, but must be unique with respect to other MAT4 or MAT5 cards.
2. If a HBDY element references this card, K is the convective film coefficient and CP is the thermal capacity per unit area.
3. MAT4 materials may be made temperature dependent by use of the MATT4 card.

Input Data Card MAT5 – Thermophysical Property Definition

Description: Defines the thermophysical properties for temperature-independent, anisotropic materials.

Format and Example:

1	2	3	4	5	6	7	8	9	10
MAT5	MID	KXX	KXY	KXZ	KYY	KYZ	KZZ	CP	
MAT5	24	0.092			0.083		0.020	0.2	

Field	Contents
MID	Material identification number (Integer > 0)
KXX,KXY,KXZ, KYY, KYZ, KZZ	Thermal conductivity (Real)
CP	Thermal capacity per unit volume (Real $\geqslant$ 0.0 or blank)

Remarks:

1. The thermal conductivity matrix has the form:

$$K = \begin{bmatrix} KXX & KXY & KXZ \\ KXY & KYY & KYZ \\ KXZ & KYZ & KZZ \end{bmatrix}$$

2. The material number may be the same as a MAT1, MAT2, or MAT3 card, but must be unique with respect to the MAT4 or MAT5 cards.

3. MAT5 materials may be made temperature dependent by use of the MATT5 card.

Input Data Card MATT4 – Thermophysical Property Temperature Dependence

Description: Specifies table reference for temperature-dependent thermal conductivity or convective film coefficient.

Format and Example:

1	2	3	4	5	6	7	8	9	10
MATT4	MID	T(K)							
MATT4	103	73							

Field	Contents
MID	Identification of a MAT4 which is to be temperature dependent (Integer > 0)
T(K)	Identification number of a TABLEMi card which gives temperature dependence of the thermal conductivity or convective film coefficient (Integer > 0 or blank)

Remarks:

1. The thermal capacity is not permitted to be temperature dependent; field 4 must be blank.
2. TABLEM1, TABLEM2, TABLEM3, or TABLEM4 type tables may be used. The basic quantity on the MAT4 card is always multiplied by the tabular function.
3. Blank or zero entries mean no table dependence of the referenced quantity on the basic MAT4 card.

Input Data Card MATT5 – Thermophysical Property Temperature Dependence

Description: Specifies table references for temperature-dependent conductivity matrix.

Format and Example:

1	2	3	4	5	6	7	8	9	10
MATT5	MID	T(KXX)	T(KXY)	T(KXZ)	T(KYY)	T(KYZ)	T(KZZ)		
MATT5	24	73							

Field	Contents
MID	Identification number of a MAT5, which is to be temperature dependent (Integer > 0)
T(K--)	Identification number of a TABLEMi card which gives temperature dependence of the matrix term (Integer > 0 or blank)

Remarks:

1. The thermal capacity is not permitted to be temperature dependent. Field 9 must be blank.
2. TABLEM1, TABLEM2, TABLEM3, or TABLEM4 type tables may be used. The basic qualities on the MAT5 card are always multiplied by the tabular function.
3. Blank or zero entries mean no table dependence of the referenced quantity on the basic MAT5 card.

Input Data Card MPC – Multipoint Constraint

Description: Defines a multipoint constraint equation of the form

$$\sum_j A_j u_j = 0$$

Format and Example:

1	2	3	4	5	6	7	8	9	10
MPC	SID	G	C	A	G	C	A		abc
MPC	3	28	1	6.2	2		4.29		+B

+bc		G	C	A	-etc.-				
+B		1	1	-2.91					

Field	Contents
SID	Set identification number (Integer > 0)
G	Identification number of grid or scalar point (Integer > 0)
C	Component number – 1 in the case of geometric grid points; blank or zero in the case of scalar points (Integer)
A	Coefficient (Real; the first A must be nonzero)

Remarks:

1. The first point in the sequence is assumed to be the dependent point and must be unique for all equations of the set.
2. Thermal powers of multipoint constraint are not recovered.
3. Multipoint constraint sets must be selected in the Case Control Deck (MPC=SID) to be used by NASTRAN Thermal Analyzer.
4. Dependent coordinates on MPC cards may not appear on ØMIT, ØMIT1, SUPØRT, SPC or SPC1 cards.
5. If a multipoint constrained grid point is part of a nonlinear element (radiative or conductive), it may only be equivalenced to another grid point.

Input Data Card NØLIN1 – Nonlinear Transient Response Dynamic Load

Description: Defines nonlinear transient forcing functions of the form

$$P_i(t) = S\,T(u_j(t))$$

Format and Example:

1	2	3	4	5	6	7	8	9	10
NØLIN1	SID	GI	CI	S	GJ	CJ	T		
NØLIN1	21	3	1	2.1	3	1	6		

Field	Contents
SID	Nonlinear load set identification number (Integer > 0)
GI	Grid or scalar or extra point identification number at which nonlinear load is to be applied (Integer > 0)
CI	Component number for GI a grid point (Integer 1); blank or zero if GI is a scalar or extra point
S	Scale factor (Real)
GJ	Grid or scalar or extra point identification number (Integer > 0)
CJ	Component number for GJ a grid point (Integer 1); blank or zero if GJ is a scalar or extra point
T	Identification number of a TABLEDi card (Integer > 0)

Remarks:

1. Nonlinear loads must be selected in the Case Control Deck (NØNLINEAR=SID) to be used by NASTRAN Thermal Analyzer.
2. Nonlinear loads may not be referenced on a DLØAD card.
3. All coordinates referenced on NØLIN1 cards must be members of the solution set. This means the u_e set for modal formulation and the $u_d = u_e + u_a$ set for direct formulation.

Input Data Card NØLIN2 – Nonlinear Transient Response Dynamic Load

Description: Defines nonlinear transient forcing functions of the form

$$P_i(t) = S\, u_j(t) u_k(t)$$

Format and Example:

1	2	3	4	5	6	7	8	9	10
NØLIN2	SID	GI	CI	S	GJ	CJ	GK	CK	
NØLIN2	14	2	1	2.9	2	1	2	1	

Field	Contents
SID	Nonlinear load set identification number (Integer > 0)
GI	Grid or scalar or extra point identification number at which nonlinear load is to be applied (Integer > 0)
CI	Component number GI a grid point (Integer 1); blank or zero if GI is a scalar or extra point
S	Scale factor (Real)
GJ	Grid or scalar or extra point identification number (Integer > 0)
CJ	Component number for GJ a grid point (Integer 1); blank or zero if GJ is a scalar or extra point
GK	Grid or scalar or extra point identification number (Integer > 0)
CK	Component number of GK a grid point (Integer 1); blank or zero if GK is a scalar or extra point

Remarks:

1. Nonlinear loads must be selected in the Case Control Deck (NØNLINEAR= SID) to be used by NASTRAN Thermal Analyzer.
2. Nonlinear loads may not be referenced on a DLØAD card.
3. All coordinates referenced on NØLIN2 cards must be members of the solution set. This means the u_e set for modal formulation and the $u_d = u_e + u_a$ set for direct formulation.

Input Data Card NØLIN3 – Nonlinear Transient Response Dynamic Load

Description: Defines nonlinear transient forcing functions of the form

$$P_i(t) = \begin{cases} S(u_j(t))^A \, ; & u_j(t) > 0 \\ 0 \, , & u_j(t) \leq 0 \end{cases}$$

Format and Example:

1	2	3	4	5	6	7	8	9	10
NØLIN3	SID	GI	CI	S	GJ	CJ	A	X	
NØLIN3	4	102		-6.1	2	1	-3.5		

Field	Contents
SID	Nonlinear load set identification number (Integer > 0)
GI	Grid or scalar or extra point identification number at which nonlinear load is to be applied (Integer > 0)
CI	Component number for GI a grid point (Integer 1); blank or zero if GI is a scalar or extra point
S	Scale factor (Real)
GJ	Grid or scalar or extra point identification number (Integer > 0)
CJ	Component number for GJ a grid point (Integer 1); blank or zero if GJ is a scalar or extra point
A	Amplification factor (Real)

Remarks:

1. Nonlinear loads must be selected in the Case Control Deck (NØNLINEAR= SID) to be used by NASTRAN Thermal Analyzer.
2. Nonlinear loads may <u>not</u> be referenced on a DLØAD card.
3. All coordinates referenced on NØLIN3 cards must be members of the solution set. This means the u_e set for modal formulation and the $u_d = u_e + u_a$ set for direct formulation.

Input Data Card NØLIN4 – Nonlinear Transient Response Dynamic Load

Description: Defines nonlinear transient forcing functions of the form

$$P_j(t) = \begin{cases} -S(-u_j(t))^A & , u_j(t) < 0 \\ 0 & , u_j(t) \geq 0 \end{cases}$$

Format and Example:

1	2	3	4	5	6	7	8	9	10
NØLIN4	SID	GI	CI	S	GJ	CJ	A		
NØLIN4	2	4	1	2.0	101		16.3		

Field	Contents
SID	Nonlinear load set identification number (Integer > 0)
GI	Grid or scalar or extra point identification number at which nonlinear load is to be applied (Integer > 0)
CI	Component number for GI a grid point (Integer 1); blank or zero if GI is a scalar or extra point
S	Scale factor (Real)
GJ	Grid or scalar or extra point identification number (Integer > 0)
CJ	Component number for GJ a grid point (Integer 1); blank or zero if GJ is a scalar or extra point
A	Amplification factor (Real)

Remarks:

1. Nonlinear loads must be selected in the Case Control Deck (NØNLINEAR= SID) to be used by NASTRAN Thermal Analyzer.
2. Nonlinear loads may not be referenced on a DLØAD card.
3. All coordinates referenced on NØLIN4 cards must be members of the solution set. This means the u_e set for modal formulation and the $u_d = u_e + u_a$ set for direct formulation.

Input Data Card ØMIT – Omitted Points

Description: Defines grid and/or scalar points that the user desires to omit from the problem through matrix partitioning.

Format and Example:

1	2	3	4	5	6	7	8	9	10
ØMIT	ID	C	ID	C	ID	C	ID	C	
ØMIT	16	1	23	0			1	0	

Field	Contents
ID	Grid or scalar point identification number (Integer > 0)
C	Component number, zero or blank for scalar points, 1 for grid points

Remarks:

1. Points specified on ØMIT cards may not be specified on ØMIT1, ASET, ASET1, SUPØRT, SPC or SPC1 cards nor may they appear as dependent points in multipoint constraint relations (MPC) or as permanent single-point constraints on GRID card.
2. As many as 4 points may be omitted by a single card.

Input Data Card ØMIT1 – Omitted Points

Description: Defines grid and/or scalar points that the user desires to omit from the problem through matrix partitioning.

Format and Example:

1	2	3	4	5	6	7	8	9	10
ØMIT1	C	G	G	G	G	G	G	G	abc
ØMIT1	1	2	1	3	10	9	6	5	ABC
+bc	G	G	G	-etc.-					
+BC	7	8							

-etc.-

Field	Contents
C	1 when point identification numbers are grid points; must be null or zero if point identification numbers are scalar points
G	Grid or scalar point identification number (Integer > 0)

Remarks: A point referenced on this card may not appear as a dependent point in a multi-point constraint relation (MPC card), nor may it be referenced on a SPC, SPC1, ØMIT, or SUPØRT card or on a GRID card as permanent single-point constraints.

Input Data Card PARAM – Parameter

Description: Specifies values for parameters used in DMAP sequences (including rigid formats).

Format and Example:

1	2	3	4	5	6	7	8	9	10
PARAM	N	V1	V2						
PARAM	HIRES	1							

Field	Contents
N	Parameter name (one to eight alphanumeric characters, the first of which is alphabetic)
V1, V2	Parameter value based on parameter type as follows:

Type	V1	V2
Integer	Integer	Blank
Real, single-precision	Real	Blank
BCD	BCD	Blank
Real, double-precision	Double-precision	Blank
Complex, single-precision	Real	Real
Complex, double-precision	Double-precision	Double-precision

Remarks:

1. Only parameters for which assigned values are allowed may be given values via the PARAM card. Section 5 of the NASTRAN User's Manual describes parameters as used in DMAP.
2. A list of parameters used in the two rigid formats of APP HEAT that may be altered by the user on PARAM cards is as follows:
   a. For the nonlinear steady-state solution (APP HEAT, SØL 3):

MAXIT (integer)	Maximum number of iterations (default 4).
EPSHT (real)	ϵ convergence parameter (default 0.001).
TABS (real)	Absolute reference temperature (default 0.0).
SIGMA (real)	Stefan-Boltzmann constant (default 0.0).
HIRES (integer)	Request residual vector output if positive (default -1).

b. For the transient solution (APP HEAT, SØL 9):

BETA (real)	Forward-differencing integration factor (default 0.55).
TABS (real)	Absolute reference temperature (default 0.0).
SIGMA (real)	Stefan–Boltzmann constant (default 0.0).
RADLIN (integer)	Radiation is linearized if possible (default -1).

Input Data Card PBAR – Simple Beam Property

Description: Defines the properties of a 1-D heat conduction element (BAR).

Format and Example:

1	2	3	4	5	6	7	8	9	10
PBAR	PID	MID	A	I1	I2	J	NSM		abc
PBAR	39	6	2.9		5.97				123
+bc	C1	C2	D1	D2	E1	E2	F1	F2	def
+23			2.0	4.0					
+ef	K1	K2	I12						

Field	Contents
PID	Property identification number (Integer > 0)
MID	Material identification number (Integer > 0)
A	Area of bar cross-section (Real)
[I1,I2,I12] *	Area moments of inertia (Real)
[J] *	Torsional constant (Real)
[NSM] *	Nonstructural mass per unit length (Real)
[K1,K2] *	Area factor for shear (Real)
[Ci,Di,Ei,Fi] *	Stress recovery coefficients (Real)

*Symbols in brackets denote those to be used only in the structural version of NASTRAN.

Input Data Card PHBDY – Property of Heat Boundary Element

Description: Defines the properties of the HBDY element.

Format and Example:

1	2	3	4	5	6	7	8	9	10
PHBDY	PID	MID	AF	E	ALPHA	R1	R2		
PHBDY	100	103	30.0	0.79					

Field	Contents
PID	Property identification number (Integer > 0)
MID	Material identification number (Integer ⩾ 0 or blank), used for convective film coefficient and thermal capacity.
AF	Area factor (Real ⩾ 0.0 or blank). Used only for HBDY types PØINT, LINE, and ELCYL.
E	Emissivity (0.0 ⩽ Real ⩽ 1.0 or blank). Used only for radiation calculations.
ALPHA	Absorptivity (0.0 ⩽ Real ⩽ 1.0 or blank). Used only for thermal vector flux calculations, default value is E.
R1,R2	"Radii" of elliptic cylinder. Used for HBDY type "ELCYL." See the HBDY element description. (Real)

Remarks:

1. The referenced material ID must be on a MAT4 card. The card defines the convective film coefficient and thermal capacity per unit area. If no material is referenced the element convection and heat capacity are zero.

2. The area factor AF is used to determine the effective area. For a "PØINT," AF = area; for "LINE" or "ELCYL," AF = effective width where area = AF · length. The effective area is automatically calculated for other HBDY types.

Input Data Card PQUAD2 – Homogeneous Quadrilateral Plate Property

Description: Defines the properties of a homogeneous 2-D quadrilateral conducting element (QUAD2).

Format and Example:

1	2	3	4	5	6	7	8	9	10
PQUAD2	PID	MID	T	NSM	PID	MID	T	NSM	
PQUAD2	32	16	2.98	9.0	45	16	5.29	6.32	

Field	Contents
PID	Property identification number (Integer > 0)
MID	Material identification number (Integer > 0)
T	Thickness (Real > 0.0)
[NSM] *	Nonstructual mass per unit area (Real)

Remarks:

1. All PQUAD2 cards must have unique identification numbers.
2. The thickness used to compute the plate properties is T.
3. One or two homogeneous quadrilateral plate properties may be defined on a single card.

*Symbols in brackets denote those to be used only in the structural version of NASTRAN.

Input Data Card PRØD – Rod Property

Description: Defines the properties of a 1-D conduction element (RØD).

Format and Example:

1	2	3	4	5	6	7	8	9	10
PRØD	PID	MID	A	J	C	NSM			
PRØD	17	23	42.6	17.92	4.236	0.5			

Field	Contents
PID	Property identification number (Integer > 0)
MID	Material identification number (Integer > 0)
A	Area of rod (Real)
[J] *	Torsional constant (Real)
[C] *	Coefficient to determine torsional stress (Real)
[NSM] *	Nonstructional mass per unit length (Real)

Remarks:

1. PRØD cards must all have unique property identification numbers.
2. For heat transfer problems, PRØD cards may only reference MAT4 or MAT5 cards.

*Symbols in brackets denote those to be used only in the structural version of NASTRAN.

Input Data Card PTRIA2 – Homogeneous Triangular Element Property

Description: Defines the properties of a homogeneous 2-D triangular conduction element (TRIA2).

Format and Example:

1	2	3	4	5	6	7	8	9	10
PTRIA2	PID	MID	T	NSM	PID	MID	T	NSM	
PTRIA2	2	16	3.92	14.7	6	16	2.96		

Field	Contents
PID	Property identification number (Integer > 0)
MID	Material identification number (Integer > 0)
T	Thickness (Real)
[NSM] *	Nonstructural mass per unit area (Real)

Remarks:

1. All PTRIA2 cards must have unique identification numbers.
2. The thickness used to compute the plate properties is T.
3. One or two homogeneous triangular element properties may be defined on a single card.

*Symbols in brackets denote those to be used only in the structural version of NASTRAN.

Input Data Card QBDY1 – Boundary Heat Flux Load

Description: Defines a uniform heat flux into HBDY elements.

Format and Example:

1	2	3	4	5	6	7	8	9	10
QBDY1	SID	QO	EID1	EID2	EID3	EID4	EID5	EID6	abc
QBDY1	109	1.-5	721						ABC

+bc	EID7	-etc.-							def
+BC									

-etc.-

Field	Contents
SID	Load set identification number (Integer > 0)
QO	Heat flux into element (Real)
EIDi	HBDY elements (Integer > 0 or "THRU")

Remarks:

1. QBDY1 cards must be selected in Case Control (LØAD = SID) to be used in the steady-state case. The thermal power into an element is given by the equation:

$$P_{in} = (\text{Effective area}) \cdot Q0.$$

2. QBDY1 cards must be referenced on a TLOADi data card for use in the transient case. The thermal power into an element is given by the equation:

$$P_{in}(t) = (\text{Effective area}) \cdot Q0 \cdot F(t - \tau),$$

where the function of time, $F(t-\tau)$, is specified on a TLØAD1 or TLØAD2 card.

3. Q0 is positive for heat input.

4. If a sequential list of elements is desired, fields 4, 5, and 6 may specify the first element, the BCD string "THRU," and the last element. No subsequent data are allowed with this option.

5. In some versions of the $\overline{\text{N}}$TA, the continuation card is not allowed.

Input Data Card QBDY2 – Boundary Heat Flux Load

Description: Defines grid point heat flux into an HBDY element.

Format and Example:

1	2	3	4	5	6	7	8	9	10
QBDY2	SID	EID	Q01	Q02	Q03	Q04			
QBDY2	109	721	1.-5	1.-5	2.-5	2.-5			

Field	Contents
SID	Load set identification number (Integer > 0)
EID	Identification number of an HBDY element (Integer > 0)
Q0i	Heat flux at the i^{th} grid point on the referenced HBDY element (Real or blank)

Remarks:

1. QBDY2 cards must be selected in Case Control (LØAD = SID) to be used in the steady-state case. The total power into each point, i, on an element is given by

$$P_i = AREA_i \cdot Q0_i.$$

2. QBDY2 cards must be referenced on a TLØAD card for use in the transient case. All connected grid points will have the same time function, but may have individual delays. The total power into each point, i, or an element is given by

$$P_{in}(t) = AREA_i \cdot Q0_i \cdot F(t - \tau_i),$$

where $F(t-\tau_i)$ is a function of time specified on a TLØAD1 or TLØAD2 card.

3. $Q0_i$ is positive for heat flux input to the element.

Input Data Card QHBDY – Heat Boundary Flux

Description: Defines a uniform heat flux into a set of grid points.

Format and Example:

1	2	3	4	5	6	7	8	9	10
QHBDY	SID	FLAG	Q0	AF	G1	G2	G3	G4	
QHBDY	120	LINE	1.5+3	0.75	13	15			

Field	Contents
SID	Load set identification number (Integer > 0)
FLAG	Type of area involved (must be one of the following "PØINT," "LINE," "REV," "AREA3," "AREA4")
Q0	Heat flux into an area (Real)
AF	Area factor depends on type (Real > 0.0 or blank)
G1,G2,G3,G4	Grid point identification of connected points (Integer > 0 or blank)

Remarks:

1. The heat flux applied to the area is transformed to loads on the points. These points need not correspond to an HBDY element.

2. The flux is applied to each grid point, i, by the equation

$$P_i = AREA_i \cdot Q0,$$

where Q0 is positive for heat input, and $AREA_i$ is the portion of the total area associated with point i.

3. In the steady-state case, the thermal load is applied with the Case Control request: LØAD = SID. In the transient case, the thermal load is applied by reference on a TLØADi data card. The load at each point will be multiplied by the function of time $F(t-\tau_i)$ defined on the TLØADi card. τ_i is the delay factor for each point.

4. The number of connected points for the five types are one(PØINT), two(LINE,REV), three(AREA3), four(AREA4). Any unused Gi entries must be on the right.

5. The area factor AF is used to determine the effective area for the PØINT and LINE types. It equals the area and the effective width, respectively. It is ignored for the other types, which have their area defined implicitly.

6. The type flag defines a surface in the same manner as the CHBDY data card. For physical descriptions of the geometry involved, see the CHBDY description.

Input Data Card QVECT – Thermal Vector Flux Load

Description: Defines thermal vector flux from a distant source into HBDY elements.

Format and Example:

1	2	3	4	5	6	7	8	9	10
QVECT	SID	Q0	E1	E2	E3	EID1	EID2	EID3	abc
QVECT	333	1.-2	-1.0	0.0	0.0	721	722	723	ABC
+bc	EID4	EID5	-etc.-						def
+BC	724								

-etc.-

Field	Contents
SID	Load set identification number (Integer > 0)
Q0	Magnitude of thermal flux vector (Real)
E1,E2,E3	Vector components (in basic coordinate system) of the thermal vector flux (Real or Integer > 0). The total flux is given by Q = Q0{E1,E2,E3}
EIDi	Element identification numbers of HBDY elements irradiated by the distant source (Integer > 0)

Remarks:

1. In the steady-state case, the load set is selected in the Case Control Deck (LØAD = SID). The total power into an element is given by

$$P_{in} = -\alpha A(e \cdot n)^* Q0,$$

where:

α = absorptivity
A = area of HBDY element
$\bar{e}$ = vector of real numbers E1,E2,E3
$\bar{n}$ = positive normal vector of element, see CHBDY data card description
$(\bar{e} \cdot \bar{n})^*$ = 0 if the vector product is positive (i.e., the flux is coming from behind the element)

2. In the transient case, the load set (SID) is selected by a TLØADi card which defines a load function of time. The total power into the element is given by

$$P_{in}(t) = -\alpha A(\bar{e}(t) \cdot \bar{n})^* Q0 \, F(t-\tau),$$

where:

α,A, and $\bar{n}$ are the same as the steady-state case

$\bar{e}(t)$ = vector of three functions of time, which may be given on TABLEDi data cards. If E1,E2, or E3 is an integer, it is the table identification number. If E1,E2 or E3 is a real number, its value is used directly; if Ei is blank, its value is zero.

$F(t-\tau)$ is a function of time specified or referenced by the parent TLØAD1 or TLØAD2 card. The value τ is calculated for each loaded point.

3. If the referenced HBDY element is of TYPE = ELCYL, the power input is an exact integration over the area exposed to the thermal flux vector.

4. If the referenced HBDY element is of TYPE = REV, the vector should be parallel to the basic z axis.

5. If a sequential list of elements is desired, fields 4, 5, and 6 may specify the first element, the BCD string "THRU," and the last element. No subsequent data are allowed with this option.

Input Data Card QVØL – Volumetric Heat Generation

Description: Defines a rate of internal heat generation in an element.

Format and Example:

1	2	3	4	5	6	7	8	9	10
QVØL	SID	QV	EID1	EID2	EID3	EID4	EID5	EID6	abc
QVØL	333	1.+2	301	302	303	317	345	416	ABC
+bc	EID7	-etc.-							def
+BC	127								

-etc.-

Field	Content
SID	Load set identification (Integer > 0)
QV	Power input per unit volume produced by a heat conduction element (Real)
EIDi	A list of heat conduction elements (Integer > 0 or BCD "THRU")

Remarks:

1. In the steady-state case, the load is applied with the Case Control request, LØAD = SID. The equivalent power into each grid point, i, connected to each element, is given by

$$P_i = QV \cdot V\emptyset L_i,$$

where $V\emptyset L_i$ is the portion of the volume associated with point i and QV is positive for heat generation.

2. In the transient case, the load is requested by reference on a TLØADi data card. The equivalent power into each grid point i is

$$P_i = QV \cdot V\emptyset L_i \cdot F(t-\tau_i),$$

where $V\emptyset L_i$ is the portion of the volume associated with point i and $F(t-\tau_i)$ is the function of time defined by a TLØADi card. τ_i is the delay for each point i.

3. If a sequential list of elements is desired, fields 4, 5, and 6 may specify the first element identification number the BCD string "THRU" and the last element identification number. No subsequent data are allowed with this option.

4. In some versions of the $\bar{N}$TA, the continuation card is not allowed.

Input Data Card RADLST – List of Radiation Areas

Description: A list of HBDY identification numbers given in the same order as the columns of the RADMTX matrix.

Format and Example:

1	2	3	4	5	6	7	8	9	10
RADLST	EID1	EID2	EID3	EID4	EID5	EID6	EID7	EID8	abc
RADLST	10	20	30	50	31	41	THRU	61	ABC

+bc	EID9	-etc.-							def
+BC	71								

-etc.-

Field	Contents
EIDi	The element identification numbers of the HBDY elements, given in the order that they appear in the RADMTX matrix (Integer > 0 or BCD "THRU")

Remarks:

1. This card is required if a RADMTX matrix (Integer > 0 or BCD "THRU")
2. Only one RADLST card string is allowed in a data deck.
3. If a group of the elements is sequential, any field except 2 and 9 may contain the BCD word "THRU". Element ID numbers will be generated for every integer between the value of the previous field and the value of the subsequent field. The values must increase, however.
4. Any element may be listed more than once. For instance, if both sides of a panel are radiating, each side may participate in a different part of the view factor matrix.

Input Data Card RADMTX – Radiation Matrix

Description: Matrix of radiation exchange coefficients for nonlinear heat transfer analysis.

Format and Example:

1	2	3	4	5	6	7	8	9	10
RADMTX	INDEX	Fi, i	Fi+1, i	Fi+2, i	Fi+3, i	Fi+4, i	Fi+5, i	Fi+6, i	abc
RADMTX	3	0	9.3	17.2	16.1	0.1	0.	6.2	ABC

+bc	Fi+7, i	-etc.-							def
+BC	6.2								

-etc.-

Field | Contents

INDEX — The column number of the matrix (Integer > 0)

Fi+k,i — The matrix values (Real), starting on the diagonal, continuing down the column. A group of zero's at the bottom of the column may be omitted. A blank field will end the column, which disallows imbedded blank fields.

Remarks:

1. The INDEX numbers go from 1 through NA, where NA is the number of radiating areas.
2. The radiation exchange coefficient matrix is symmetric, and only the lower triangle is input. Column 1 is associated with the HBDY element first listed on the RADLST card, Column 2 for the next, etc. Null columns need not be entered.
3. $$P_i = \sum_{j=1}^{NA} F_{ij} q_j$$

   P_i = total irradiation into element i

   q_j = radiosity (per unit area) at j

   F_{ij} = radiation matrix (units of area, a product of the view factor and the radiating area)
4. A column may only be specified once.

Input Data Card SEQGP – Grid and Scalar Point Resequencing

Description: Used to order the grid points and user-supplied scalar points of the problem. The purpose of this card is to allow the user to reidentify the formation sequence of the grid and scalar points of the model in such a way as to optimize bandwidth which is essential for efficient solutions.

Format and Example:

1	2	3	4	5	6	7	8	9	10
SEQGP	ID	SEQID	ID	SEQID	ID	SEQID	ID	SEQID	
SEQGP	5392	15.6			2	1.9.2.6	3	2	

Field	Contents
ID	Grid or scalar point identification number (Integer > 0)
SEQID	Sequenced identification number (a special number described below)

Remarks:

1. ID is any grid or scalar point identification number which is to be reidentified for sequencing purposes. The grid point sequence number (SEQID) is a special number which may have any of the following forms where X is a decimal integer digit - XXX.X.X.X,XXXX.X.X, XXX.X or XXXX where any of the leading X's may be omitted. This number must contain no imbedded blanks.

2. If the user wishes to insert a grid point between two already existing grid points, such as 15 and 16, for example, he would define it as, say 5392, and then use this card to insert grid point number 5392 between them by equivalencing it to, say 15.6. All output referencing this point will refer to 5392.

3. The SEQID numbers must be unique and may not be the same as a point ID which is not being changed. No grid point ID may be referenced more than once.

4. No continuation cards (small field or large field) are allowed with either the SEQGP or the SEQEP card.

5. From one to four grid or scalar points may be resequenced on a single card.

Input Data Card SLØAD – Static Thermal Load

Description: Used to apply static thermal loads to scalar or grid points.

Format and Example:

1	2	3	4	5	6	7	8	9	10
SLØAD	SID	S	F	S	F	S	F		
SLØAD	16	2	5.9	17	-6.3	14	-2.93		

Field	Contents
SID	Load set identification number (Integer > 0)
S	Scalar or grid point identification number (Integer > 0)
F	Load value (Real)

Remarks:

1. Load sets must be selected in the Case Control Deck (LØAD=SID) to be used by NASTRAN Thermal Analyzer.
2. Up to three different thermal loads may be defined on a single card.
3. This card may be used in all three heat transfer rigid formats, subject to their respective rules.

Input Data Card SPC – Single-Point Constraint

Description: Defines sets of single-point constraints and prescribed temperatures.

Format and Example:

1	2	3	4	5	6	7	8	9	10
SPC	SID	G	C	D	G	C	D		
SPC	2	32	1	-2.6	5	0	+2.9		

Field	Contents
SID	Identification number of single-point constraint set (Integer > 0)
G	Grid or scalar point identification number (Integer > 0)
C	Component number (1 for grid points and 0 for scalar points)
D	Value of prescribed temperature for the designated point (Real)

Remarks:

1. A point referenced on this card may not appear as a dependent point in a multipoint constraint relation (MPC card), nor may it be referenced on a SPC1, ØMIT, ØMIT1 or SUPØRT card. D must be 0.0 for transient problems.
2. Single-point forces of constraint are recovered during heat flux data recovery.
3. Single-point constraint sets must be selected in the Case Control Deck (SPC=SID) to be used by NASTRAN Thermal Analyzer.
4. From one to two single-point constraints may be defined on a single card.
5. SPC degrees of freedom may be redundantly specified as permanent constraints on the GRID card.
6. Use this card for APP HEAT SØL 1 only (some versions of the NTA will produce incorrect answers if this card is used in SØL 3).

Input Data Card SPC1 – Single-Point Constraint

Description: Defines sets of single-point constraints.

Format and Example:

1	2	3	4	5	6	7	8	9	10
SPC1	SID	C	G1	G2	G3	G4	G5	G6	abc
SPC1	3	1	1	3	10	9	6	5	ABC

+bc	G7	G8	G9	-etc.-					
+BC	2	8							

Alternate Form

SPC1	SID	C	GID1	"THRU"	GID2				
SPC1	313	1	6	THRU	32				

Field	Contents
SID	Identification number of single-point constraint set (Integer > 0)
C	Component number (1 for grid points, 0 for scalar points)
Gi, GIDi	Grid or scalar point identification numbers (Integer > 0)

Remarks:

1. Note that prescribed temperatures are not available via this card. As many continuation cards as desired may appear when "THRU" is not used.
2. A point referenced on this card may not appear as a dependent point in a multipoint constraint relation, nor may it be referenced on a SPC, ØMIT, ØMIT1, SUPØRT card.
3. Single-point constraint sets must be selected in the Case Control Deck (SPC=SID) to be used by NASTRAN Thermal Analyzer.
4. SPC degrees of freedom may be redundantly specified as permanent constraints on the GRID card.
5. All grid points referenced by GID1 thru GID2 must exist.
6. Use this card for APP HEAT SØL 3 to identify those grid points where temperatures are prescribed but the values of temperature are given on the TEMP or TEMPD cards.

Input Data Card SPØINT – Scalar Point

Description: Defines scalar points of the thermal model.

Format and Example:

1	2	3	4	5	6	7	8	9	10
SPØINT	ID	ID	ID	ID	ID	ID	ID	ID	
SPØINT	3	18	1	4	16	2			

Alternate Form

SPØINT	ID1	"THRU"	ID2						
SPOINT	5	THRU	649						

Field — Contents

ID, ID1, ID2 — Scalar point identification number (Integer > 0; ID1 < ID2)

Remarks:

1. Scalar points defined by their appearance on a scalar connection card need not appear on a SPØINT card.
2. Identification numbers of scalar points, extra points (EPØINT) and geometric grid points (GRID) must all be unique.
3. This card is used primarily to define scalar points appearing in single or multipoint constraint equations but to which no scalar elements are connected.
4. If the alternate form is used, scalar points ID1 through ID2 are defined.
5. For a discussion of scalar points, see section 5.6 of the NASTRAN Theoretical Manual.

Input Data Card TABLED1 – Transient (dynamic) Thermal Load Tabular Function

Description: Defines a tabular function for use in generating time-dependent transient thermal loads.

Format and Example:

1	2	3	4	5	6	7	8	9	10
TABLED1	ID								+abc
TABLED1	32								ABC

+abc	x_1	y_1	x_2	y_2	x_3	y_3	x_4	y_4	+def
+BC	-3.0	6.9	2.0	5.6	3.0	5.6	ENDT		

-etc.-

Field	Contents
ID	Table identification number (Integer > 0)
x_i, y_i	Tabular entries (Real)

Remarks:

1. The x_i must be in either ascending or descending order but not both.
2. Jumps between two points ($x_i = x_{i+1}$) are allowed, but not at the end points.
3. At least two entries must be present.
4. Any x–y entry may be ignored by placing the BCD string "SKIP" in either of the two fields used for that entry.
5. The end of the table is indicated by the existence of the BCD string "ENDT" in either of the two fields following the last entry. An error is detected if any continuation cards follow the card containing the end-of-table flag "ENDT."
6. Each TABLEDi mnemonic infers the use of a specific algorithm. For TABLED1 type tables, this algorithm is

$$\boxed{Y = y_T(X)}$$

where X is input to the table and Y is returned. The table look-up $y_T(x)$, $x = X$, is performed using linear interpolation within the table and linear extrapolation outside the table using the last two end points at the appropriate table end. At jump points the average $y_T(x)$ is used. There are no error returns from this table look-up procedure.

Input Data Card TABLED2 – Transient (dynamic) Thermal Load Tabular Function

Description: Defines a tabular function for use in generating time–dependent transient thermal loads. Also contains parametric data for use with the table.

Format and Example:

1	2	3	4	5	6	7	8	9	10
TABLED2	ID	X1							+abc
TABLED2	15	-10.5							ABC

+abc	x_1	y_1	x_2	y_2	x_3	y_3	x_4	y_4	+def
+BC	1.0	-4.5	2.0	-4.2	2.0	2.8	7.0	6.5	DEF

+def	x_5	y_5	x_6	y_6	x_7	y_7	x_8	y_8	+ghi
+EF	SKIP	SKIP	9.0	6.5	ENDT				

-etc.-

Field	Contents
ID	Table identification number (Integer > 0)
X1	Table parameter (Real)
x_i, y_i	Tabular entries (Real)

Remarks:

1. The x_i must be in either ascending or descending order but not both.
2. Jumps between two points ($x_i = x_{i+1}$) are allowed, but not at the end points.
3. At least two entries must be present.
4. Any x–y entry may be ignored by placing the BCD string "SKIP" in either of the two fields used for that entry.
5. The end of the table is indicated by the existence of the BCD string "ENDT" in either of the two fields following the last entry. An error is detected if any continuation cards follow the card containing the end–of–table flag "ENDT."
6. Each TABLEDi mnemonic infers the use of a specific algorithm. For TABLED2 type tables, this algorithm is

$$\boxed{Y = y_T(X - X1)}$$

where X is input to the table and Y is returned. The table look-up $y_T(x)$, $x = X-X1$, is performed using linear interpolation within the table and linear extrapolation outside the table using the last two end points at the appropriate table end. At jump points the average $y_T(x)$ is used. There are no error returns from this table look-up procedure.

Input Data Card TABLED3 – Transient (dynamic) Thermal Load Tabular Function

Description: Defines a tabular function for use in generating time-dependent transient thermal loads. Also contains parametric data for use with the table.

Format and Example:

1	2	3	4	5	6	7	8	9	10
TABLED3	ID	X1	X2						+abc
TABLED3	62	126.9	30.0						ABC

+abc	x_1	y_1	x_2	y_2	x_3	y_3	x_4	y_4	+def
+BC	2.9	2.9	3.6	4.7	5.2	5.7	ENDT		

-etc.-

Field	Contents
ID	Table identification number (Integer > 0)
X1, X2	Table parameters (Real; X2 ≠ 0.0)
x_i, y_i	Tabular entries (Real)

Remarks:

1. The x_i must be in either ascending or descending order but not both.
2. Jumps between two points ($x_i = x_{i+1}$) are allowed, but not at the end points.
3. At least two entries must be present.
4. Any x-y entry may be ignored by placing the BCD string "SKIP" in either of the two fields used for that entry.
5. The end of the table is indicated by the existence of the BCD string "ENDT" in either of the two fields following the last entry. An error is detected if any continuation cards follow the card containing the end-of-table flag "ENDT."
6. Each TABLEDi mnemonic infers the use of a specific algorithm. For TABLED3 type tables, this algorithm is

$$\boxed{Y = y_T\left(\frac{X - X1}{X2}\right)}$$

where X is input to the table and Y is returned. The table look-up $y_T(x)$,

$$x = \frac{X - X1}{X2},$$

is performed using linear interpolation within the table and linear extrapolation outside the table using the last two end points at the appropriate table end. At jump points the average $y_T(x)$ is used. There are no error returns from this table look-up procedure.

Input Data Card TABLED4 – Transient (dynamic) Thermal Load Tabular Function

Description: Defines coefficients of a power series for use in generating time-dependent transient thermal loads. Also contains parametric data for use with the table.

Format and Example:

1	2	3	4	5	6	7	8	9	10
TABLED4	ID	X1	X2	X3	X4				+abc
TABLED4	28	0.0	1.0	0.0	100.				ABC

+abc	A_0	A_1	A_2	A_3	A_4	A_5	A_6	A_7	+def
+BC	2.91	-0.0329	6.51-5	0.0	-3.4-7	ENDT			

-etc.-

Field	Contents
ID	Table identification number (Integer > 0)
X1, X2, X3, X4	Table parameters (Real; $X2 \neq 0.0$; $X3 < X4$)
A_i	Coefficient entries (Real)

Remarks:

1. At least one entry must be present.
2. The end of the table is indicated by the existence of the BCD string "ENDT" in the field following the last entry. An error is detected if any continuation cards follow the card containing the end-of-table flag "ENDT."
3. Each TABLEDi mnemonic infers the use of a specific algorithm. For TABLED4 type tables, this algorithm is

$$\boxed{Y = \sum_{i=0}^{N} A_i \left(\frac{X - X1}{X2}\right)^i}$$

where X is input to the table and Y is returned. Whenever $X < X3$, use X3 for X; whenever $X > X4$, use X4 for X. There are $N + 1$ entries in the table. There are no error returns from this table look-up procedure.

Input Data Card TABLEM1 – Material Property Table

Description: Defines a tabular function for use in generating temperature-dependent material properties.

Format and Example:

1	2	3	4	5	6	7	8	9	10
TABLEM1	ID								+abc
TABLEM1	32								ABC
+abc	x_1	y_1	x_2	y_2	x_3	y_3	x_4	y_4	+def
+BC	-3.0	6.9	2.0	5.6	3.0	5.6	ENDT		

-etc.-

Field	Contents
ID	Table identification number (Integer > 0)
x_i, y_i	Tabular entries (Real)

Remarks:

1. The x_i must be in either ascending or descending order but not both.
2. Jumps between two points ($x_1 = x_{i+1}$) are allowed, but not at the end points.
3. At least two entries must be present.
4. Any x–y entry may be ignored by placing the BCD string "SKIP" in either of the two fields used for that entry.
5. The end of the table is indicated by the existence of the BCD string "ENDT" in either of the two fields following the last entry. An error is detected if any continuation cards follow the card containing the end–of–table flag "ENDT."
6. Each TABLEMi mnemonic infers the use of a specific algorithm. For TABLEM1 type tables, this algorithm is

$$\boxed{Y = y_T(X)}$$

where X is input to the table and Y is returned. The table look–up $y_T(x)$, x = X, is performed using linear interpolation within the table and linear extrapolation outside the table using the last two end points at the appropriate table end. At jump points the average $y_T(x)$ is used. There are no error returns from this table look–up procedure.

Input Data Card TABLEM2 – Material Property Table

Description: Defines a tabular function for use in generating temperature-dependent material properties. Also contains parametric data for use with the table.

Format and Example:

1	2	3	4	5	6	7	8	9	10
TABLEM2	ID	X1							+abc
TABLEM2	15	-10.5							ABC
+abc	x_1	y_1	x_2	y_2	x_3	y_3	x_4	y_4	+def
+BC	1.0	-4.5	2.0	-4.5	2.0	2.8	7.0	6.5	DEF
+def	x_5	y_5	x_6	y_6	x_7	y_7	x_8	y_8	+ghi
+EF	SKIP	SKIP	9.0	6.5	ENDT				

-etc.-

Field	Contents
ID	Table identification number (Integer > 0)
X1	Table parameter (Real)
x_i, y_i	Tabular entries (Real)

Remarks:

1. The x_i must be in either ascending or descending order but not both.
2. Jumps between two points ($x_i = x_{i+1}$) are allowed, but not at the end points.
3. At least two entries must be present.
4. Any x–y entry may be ignored by placing the BCD string "SKIP" in either of the two fields used for that entry.
5. The end of the table is indicated by the existence of the BCD string "ENDT" in either of the two fields following the last entry. An error is detected if any continuation cards follow the card containing the end-of-table flag "ENDT."
6. Each TABLEMi mnemonic infers the use of a specific algorithm. For TABLEM2 type tables, this algorithm is

$$\boxed{Y = Z\, y_T\,(X - X1)}$$

where X is input to the table, Y is returned and Z is supplied from the basic MATi card. The table look-up $y_T(x)$, $x = X - X1$, is performed using linear interpolation within the table and linear extrapolation outside the table using the last two end points at the appropriate table end. At jump points the average $y_T(x)$ is used. There are no error returns from this table look-up procedure.

Input Data Card TABLEM3 – Material Property Table

Description: Defines a tabular function for use in generating temperature-dependent material properties. Also contains parametric data for use with the table.

Format and Example:

1	2	3	4	5	6	7	8	9	10
TABLEM3	ID	X1	X2						+abc
TABLEM3	62	126.9	30.0						ABC

+abc	x_1	y_1	x_2	y_2	x_3	y_3	x_4	y_4	+def
+BC	2.9	2.9	3.6	4.7	5.2	5.7	ENDT		

-etc.-

Field	Contents
ID	Table identification number (Integer > 0)
X1, X2	Table parameters (Real; X2 $\neq$ 0.0)
x_i, y_i	Tabular entries (Real)

Remarks:

1. The x_i must be in either ascending or descending order but not both.
2. Jumps between two points ($x_i = x_{i+1}$) are allowed, but not at the end points.
3. At least two entries must be present.
4. Any x–y entry may be ignored by placing the BCD string "SKIP" in either of the two fields used for that entry.
5. The end of the table is indicated by the existence of the BCD string "ENDT" in either of the two fields following the last entry. An error is detected if any continuation cards follow the card containing the end–of–table flag "ENDT."
6. Each TABLEMi mnemonic infers the use of a specific algorithm. For TABLEM3 type tables, this algorithm is

$$\boxed{Y = Z\, y_T\left(\frac{X - X1}{X2}\right)}$$

where X is input to the table, Y is returned and Z is supplied from basic MATi card. The table look-up $y_T(x)$,

$$x = \frac{X - X1}{X2},$$

is performed using linear interpolation within the table and linear extrapolation outside the table using the last two end points at the appropriate table end. At jump points the average $y_T(x)$ is used. There are no error returns from this table look-up procedure.

Input Data Card TABLEM4 – Material Property Table

Description: Defines coefficients of a power series for use in generating temperature-dependent material properties. Also contains parametric data for use with the table.

Format and Example:

1	2	3	4	5	6	7	8	9	10
TABLEM4	ID	X1	X2	X3	X4				+abc
TABLEM4	28	0.0	1.0	0.0	100.				ABC

+abc	A_0	A_1	A_2	A_3	A_4	A_5	A_6	A_7	+def
+BC	2.91	-0.0329	6.51-5	0.0	-3.4-7	ENDT			

-etc.-

Field	Contents
ID	Table identification number (Integer > 0)
X1, X2, X3, X4	Table parameters (Real; X2 ≠ 0.0; X3 < X4)
A_i	Coefficient entries (Real)

Remarks:

1. At least one entry must be present.
2. The end of the table is indicated by the existence of the BCD string "ENDT" in the field following the last entry. An error is detected if any continuation cards follow the card containing the end-of-table flag "ENDT."
3. Each TABLEMi mnemonic infers the use of a specific algorithm. For TABLEM4 type tables, this algorithm is

$$\boxed{Y = Z \sum_{i=0}^{N} A_i \left(\frac{X - X1}{X2}\right)^i}$$

where X is input to the table, Y is returned and Z is supplied from the basic MATi card. Whenever X < X3, use X3 for X; whenever X > X4, use X4 for X. There are N +1 entries in the table. There are no error returns from this table look-up procedure.

Input Data Card TEMP – Point Temperatures

Description: Defines temperature at grid points, scalar points and/or extra points.

Format and Example:

1	2	3	4	5	6	7	8	9	10
TEMP	SID	G	T	G	T	G	T		
TEMP	3	94	316.2	49	219.8				

Field	Contents
SID	Temperature set identification number (Integer > 0)
G	Point identification number (Integer > 0)
T	Temperature (Real)

Remarks:

1. Temperature sets must be selected in the Case Control Deck (TEMP=SID) to be used by NASTRAN Thermal Analyzer.

2. From one to three point temperatures may be defined on a single card.

3. If the element material is temperature dependent, its properties are evaluated at the average temperature.

4. Average element temperatures are obtained as a simple average of the connecting grid point temperatures when no element temperature data are defined.

5.* For APP HEAT SØL 3 and SØL 9, select TEMP set in Case Control with TEMP (MATERIAL) = SET ID to define the vector (estimate of final temperature result).

6. For APP HEAT SØL 9, the TEMP set will define the initial conditions when selected in Case Control by IC = SET ID.

*For nonlinear runs using SØL3, it is critical that this guess vector be higher than the actual steady-state result to ensure solution convergence.

Input Data Card TEMPD – Point Temperature Default

Description: Defines a temperature default for all points of the thermal model which have not been given a temperature on a TEMP card.

Format and Example:

1	2	3	4	5	6	7	8	9	10
TEMPD	SID	T	SID	T	SID	T	SID	T	
TEMPD	1	216.3							

Field	Contents
SID	Temperature set identification number (Integer > 0)
T	Default temperature (Real)

Remarks:

1. Temperature sets must be selected in the Case Control Deck (TEMP=SID) to be used by NASTRAN Thermal Analyzer.
2. From one to four default temperatures may be defined on a single card. Some versions of the NTA contain an error which may be avoided by specifying only one default temperature set per TEMPD card (i.e., use only fields 1, 2, and 3).
3. If the element material is temperature dependent, its properties are evaluated at the average temperature.
4. Average element temperatures are obtained as a simple average of the connecting grid point temperatures when no element temperature data are defined.
5. See TEMP card for additional comments.

Input Data Card TF – Dynamic Transfer Function

Description: 1. May be used to define a transfer function of the form

$$(B0 + B1p + B2p^2)u_d + \sum_i (A0(i) + A1(i)p + A2(i)p^2)u_i = 0$$

2. May be used as a means of direct matrix input.

Format and Example:

1	2	3	4	5	6	7	8	9	10
TF	SID	GD	CD	B0	B1	B2			+abc
TF	1	2	3	4.0	5.0	6.0			+ABC

+abc	G(1)	C(1)	A0(1)	A1(1)	A2(1)				+def
+ABC	3	4	5.0	6.0	7.0				+DEF

-etc.-

Field	Contents
SID	Set identification number (Integer > 0)
GD,G(i)	Grid, scalar or extra point identification numbers (Integer > 0)
CD, C(i)	Component numbers (Null or zero for scalar or extra points, 1 for a grid point)
B0,B1,B2 A0(i),A1(i), A2(i)	Transfer function coefficients (Real)

Remarks:

1. The matrix elements defined by this card are added to the dynamic matrices for the problem.
2. Transfer Function sets must be selected in the Case Control Deck (TFL= SID) to be used by NASTRAN Thermal Analyzer.
3. The constraint relation given above will hold only if no elements are connected to the dependent coordinate.

Input Data Card TIC – Transient Initial Condition

Description: Defines values for the initial conditions of transient thermal analysis.

Format and Example:

1	2	3	4	5	6	7	8	9	10
TIC	SID	G	C	UO	VO				
TIC	1	3	1	5.0	-6.0				

Field	Contents
SID	Set identification number (Integer > 0)
G	Grid or scalar or extra point identification number (Integer > 0)
C	Component number (Null or zero for scalar or extra points, 1 for a grid point)
UO	Initial temperature value (Real)
[VO] *	Initial velocity value (Real)

Remarks:

1. Transient initial condition sets must be selected in the Case Control Deck (IC=SID) to be used by NASTRAN Thermal Analyzer.
2. If no TIC set is selected in Case Control Deck, all initial conditions are assumed zero.
3. Initial conditions for temperatures not specified on TIC cards will be assumed zero.
4. TEMP and TEMPD sets are usually used in place of TIC sets in defining initial temperatures for a transient solution.

*Symbols in brackets denote those to be used only in the structural version of NASTRAN.

Input Data Card TLØAD1 – Transient Thermal Load

Description: Defines a time-dependent transient thermal load of the form

$$\{P(t)\} = \{A\,F(t-\tau)\}$$

for use in transient response problems.

Format and Example:

1	2	3	4	5	6	7	8	9	10
TLØAD1	SID	L	M		TF				
TLØAD1	5	7	9		13				

Field	Contents
SID	Set identification number (Integer > 0)
L	Identification number of DAREA card set or a thermal load set (QBDYi, QHBDY, QVECT, and QVØL) which defines A (Integer > 0)
M	Identification number of DELAY card set which defines τ (Integer $\geq$ 0)
TF	Identification number of TABLEDi card which gives $F(t-\tau)$ (Integer > 0)

Remarks:

1. If M is zero, τ will be zero.
2. Field 5 must be blank.
3. Dynamic load sets must be selected in the Case Control Deck (DLØAD= SID) to be used by NASTRAN thermal analyzer.
4. TLØAD1 loads may be combined with TLØAD2 loads only by specification on a DLØAD card. That is, the SID on a TLØAD1 card may not be the same as that on a TLØAD2 card.
5. SID must be unique for all TLØAD1 and TLØAD2 cards.
6. A referenced QVECT data card may also contain references to functions of time, and therefore A may be a function of time.

Input Data Card TLØAD2 – Transient Thermal Load

Description: Defines a time–dependent transient thermal load of the form

$$\{P(t)\} = \begin{cases} \{0\}, \ \tilde{t} < 0 \text{ or } \tilde{t} > (T2 - T1) \\ \{A \ \tilde{t}^{B} \ e^{C\tilde{t}} \cos(2\pi F\tilde{t} + P)\}, 0 \leq \tilde{t} \leq (T2 - T1) \end{cases}$$

for use in transient thermal problems where $\tilde{t} = t - T1 - \tau$.

Format and Example:

1	2	3	4	5	6	7	8	9	10
TLØAD2	SID	L	M		T1	T2	F	P	abc
TLØAD2	4	10	7		2.1	4.7	12.0	30.0	+12

+bc	C	B							
+12	2.0	3.0							

Field	Contents
SID	Set identification number (Integer > 0)
L	Identification number of DAREA card set or a thermal load set (QBDYi, QHBDY, QVECT, and QVØL) which defines A (Integer > 0)
M	Identification number of DELAY card set which defines τ (Integer ≥ 0)
T1	Time constant (Real ≥ 0.0)
T2	Time constant (Real, T2 > T1)
F	Frequency in cycles per unit time (Real ≥ 0.0)
P	Phase angle in degrees (Real)
C	Exponential coefficient (Real)
B	Growth coefficient (Real)

Remarks:

1. If M is zero, τ will be zero.
2. Field 5 must be blank.
3. Transient thermal load sets must be selected in the Case Control Deck (DLØAD=SID) to be used by NASTRAN Thermal Analyzer.

4. TLØAD2 loads may be combined with TLØAD1 loads only by specification on a DLØAD card. That is, the SID on a TLØAD2 card may not be the same as that on a TLØAD1 card.
5. SID must be unique for all TLØAD1 and TLØAD2 cards.
6. A referenced QVECT load card may also contain references to functions of time, and therefore A may be a function of time.

Input Data Card TSTEP – Transient Time Step

Description: Defines time step intervals at which solutions will be generated and output in transient thermal analysis.

Format and Example:

1	2	3	4	5	6	7	8	9	10
TSTEP	SID	N(1)	DT(1)	NØ(1)					+abc
TSTEP	2	10	0.001	5					+ABC
+abc		N(2)	DT(2)	NØ(2)					+def
+ABC		9	0.01	1					+DEF

-etc.-

Field	Contents
SID	Set identification number (Integer > 0)
N(i)	Number of time steps of value DT(i) (Integer ≥ 2)
DT(i)	Time increment (Real > 0.0)
NØ(i)	Skip factor for output (Every NØ(i)$^{\underline{th}}$ step will be saved for output) (Integer > 0)

Remarks: TSTEP cards must be selected in the Case Control Deck (TSTEP=SID) in order to be used by NASTRAN Thermal Analyzer.

REFERENCES

1. Lee, H. P., and Mason, J. B., *NASTRAN Thermal Analyzer: A General Purpose Finite-Element Heat Transfer Computer Program,* The 2nd NASTRAN User's Colloquium, NASA TMX-2637, September 1972, pp. 443-454.

2. Butler, T. G., and Michel, D., *NASTRAN: A Summary of the Functions and Capabilities of the NASA Structural Analysis Computer System,* NASA SP-260, 1971, Washington, D.C.

3. MacNeal, R. H. (Editor), *The NASTRAN Theoretical Manual,* NASA SP-221, with Level 15.5 updates, December 1972, Washington, D.C.

4. McCormick, C. W. (Editor), *The NASTRAN User's Manual (Level 15),* NASA SP-222(01), with Level 15.5 updates, June 1972, Washington, D.C.

5. *The NASTRAN Programmer's Manual,* NASA SP-223(01), with Level 15.5 updates, May 1973, Washington, D.C.

6. Turner, J. L., Clough, R. W., Martin, H. C., and Topp, L. J., *Stiffness and Deflection Analysis of Complex Structures,* Journal of Aerospace Science, Vol. 23, No. 9, September 1956, pp. 805–825.

7. Zienkiewicz, O. C. and Cheung, Y. K., *Finite Elements in the Solution of Field Problems,* The Engineer, Vol. 220, September 1965, pp. 507–510.

8. Visser, W., *A Finite Element Method for the Determination of Non-Stationary Temperature Distribution and Thermal Deformations,* Proceedings of Conference on Matrix Methods in Structural Mechanics, AFFDL-TR-66-80, December 1965, pp. 925–943.

9. Hildebrand, F. B., *Methods of Applied Mathematics,* Prentice-Hall, Inc., New York, 1954, pp. 120–226.

10. Kantorovich, L. V. and Krylov, V. I., *Approximate Methods of Higher Analysis,* translated by C. D. Benster, Interscience Publishers, Inc., New York, 1958, pp. 241–303.

11. Zienkiewicz, O. C. and Parekh, C. J., *Transient Field Problems: Two-Dimensional and Three-Dimensional Analysis by Isoparametric,* International Journal for Numerical Methods in Engineering, Vol. 2, pp. 61–71, 1970.

12. Guyan, R. L., *Reduction of Stiffness and Mass Matrices,* AIAA Journal, Vol. 3, No. 2, February 1965, pp. 380.

13. Scarborough, J. B., *Numerical Mathematical Analysis,* 4th Edition, The Johns Hopkins Press, Baltimore, 1958, pp. 192–211.

14. Newmark, N. M., *A Method of Computation for Structural Dynamics,* Proc. ASCE, Journal of the Engineering Mechanics Div., EM–3, July 1959, pp. 67–94.

15. Jackson, C. E., Jr., *NASTRAN Thermal Analyzer—Theory and Application Including A Guide to Modeling Engineering Problems, Volume II: A Guide to the GSFC NTA Sample Problem Library,* NASA-GSFC, X–322–76–17, December 1975.

16. Lee, H. P., *Structural-Thermal-Optical Program (STOP),* NASA-GSFC 1970 Science and Technology Review. NASA-Goddard Space Flight Center, January 13–14, 1971; also *Significant Accomplishments in Technology: Goddard Space Flight Center, 1970,* NASA SP–295, 1972, Washington, D.C., pp. 36–40.

17. Thermophysics Branch, *Nodal Network Thermal Balance Program,* NASA Goddard Space Flight Center, February 1968.

18. Wilson, E. L., and Nickell, R. E., *Application of the Finite Element Method of Heat Conduction Analysis,* Nuclear Engineering and Design, Vol. 4, 1966, pp. 276–284.

19. Gurtin, M. E., *Variational Principles for Linear Initial-Value Problems,* Quarterly Applied Mathematics, Vol. 22, No. 3, 1964, pp. 252–256.

20. Brocci, R. A., *Analysis of Axisymmetric Linear Heat Conduction Problems by Finite Element Method,* ASME Paper 69–WA/HT–37, Winter Annual Meeting, Los Angeles, California, 1969.

21. Aguirre-Ramirez, G., and Oden, J. T., *Finite Element Technique Applied to Heat Conduction in Solid with Temperature Dependent Thermal Conductivity,* ASME Paper 69–WA/HT–34, Winter Annual Meeting, Los Angeles, California, 1969.

22. Rybicki, E. F., and Hopper, A. T., *Higher Order Element Method for Transient Temperature Analysis of Inhomogeneous Materials,* ASME Paper 69–WA/HT–33, Winter Annual Meeting, Los Angeles, California, 1969.

23. Gallagher, R. H., and Mallett, R.: *Efficient Solution Processes for Finite Element Analysis of Transient Heat Conduction,* ASME Paper 69–WA/HT–32, also ASME-Journal of Heat Transfer, Vol. 93, No. 3, August 1971, pp. 257–263.

24. Fujino, T., and Ohsaka, K., *The Heat Conduction and Thermal Stress Analysis by the Finite Element Method,* Proceedings of the Second Conference on Matrix Method in Structural Mechanics, AFFDL-TR-68-150, Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio, December 1969, pp. 1121-1163.

25. Becker, E. B., and Parr, C. H., *Application of the Finite Element Method to Heat Conduction in Solids,* T. R. S-117, Rohm and Haas Co., Philadelphia, Pa., 1967.

26. Chaloupka, A. B., *A Computer Program for the Analysis of Two-Dimensional Heat Conduction Using the Finite Element Technique,* AD-690450, M. S. thesis, Naval Post-graduate School, Monterey, California, June 6, 1969.

27. Mote, C. D., Jr., *Unsymmetrical, Transient Heat Conduction: Rotating Disk Applications,* ASME-Journal of Engineering for Industry, Vol. 92, No. 1, February 1970, pp. 181-190.

28. Emery, A. F., and Carson, W. W., *An Evaluation of the Use of the Finite-Element Method in the Computation of Temperature,* ASME Paper 69-WA/HT-38, also ASME-Journal of Heat Transfer, May, 1971, Vol. 93, No. 2, pp. 136-145.

29. Lemmon, E. C., and Heaton, H. S., *Accuracy, Stability and Oscillation Characteristics of Finite Element Method for Solving Heat Conduction Equation,* ASME Paper 69-WA/HT-35, Winter Annual Meeting, Los Angeles, California, 1969.

30. Richardson, P. D., and Shum, Y. M., *Use of Finite Element Methods in Solution of Transient Heat Transfer Conduction Problems,* ASME Paper 69-WA/HT-36, Winter Annual Meeting, Los Angeles, California, 1969.

31. Lee, H. P., *Application of Finite-Element Method in the Computation of Temperature with Emphasis on Radiative Exchanges,* Progress in Astronautics and Aeronautics, Vol. 31 – Thermal Control and Radiation, MIT Press, 1973, pp. 491-520.

32. Heuser, J. S., *Finite Element Method for Thermal Analysis,* NASA-GSFC Test and Evaluation Division, Program STOP-Document, Greenbelt, Maryland, 1971.

33. Mason, J. B., *The Solution of Heat Transfer Problems by the Finite Element Method Using NASTRAN,* NASA-GSFC, Test and Evaluation Div. Rept. X-321-70-97, Greenbelt, Maryland, 1970.

34. Lee, H. P., and Jackson, C. E., Jr., *Thermal Analysis of the Telescope of the OSO-I Colorado Experiment,* NASA-GSFC Test and Evaluation Division, Program STOP-Document, Greenbelt, Maryland, August 1973.

35. Lee, H. P., and Jackson, C. E., Jr.: Using the NASTRAN Thermal Analyzer to Simulate a Flight Scientific Instrument Package, Proceedings of the 20th Annual Meeting of the Institute of Environmental Sciences, Washington, D.C., April 1974, pp. 152-158.

36. Jackson, C. E., Jr., *A Summary of the NASTRAN Thermal Analyses Performed on the IUE Scientific Instrument From 1-1-74 to 7-23-75,* NASA-GSFC, Test and Evaluation Div., X-322-75-306, August 1975.

37. Dusinberre, G. M., *Heat Transfer Calculations by Finite Differences,* International Text Book Co., Scranton, Pa., 1961.

38. Puccinelli, E. F., *View Factor Computer Program (Program VIEW) User's Manual,* NASA-GSFC, Test and Evaluation Div., X-324-73-272, July, 1973.

39. Jackson, C. E., Jr., *Programmer's Manual for VIEW – A Modification of the RAVFAC View Factor Program for Use with the NASTRAN Thermal Analyzer on IBM-360 Series Computers,* NASA-GSFC Test and Evaluation Div., X-322-73-120, March 1973.

40. Baker, A. J., and Manhardt, P. D.: Finite Element Solution for Energy Conservation Using a Highly Stable Explicit Integration Algorithm, NASA CR–130149, Bell Aerospace Company, October 1972.

41. Harder, R. L.: NASTRAN Variance Analysis and Plotting of HBDY Elements, NASA CR–139007, The MacNeal–Schwendler Corp., May 1974.

42. Lee, H. P., and Jackson, C. E., Jr.: Finite–Element Solution for a Combined Radiative-Conductive Analysis with Mixed Diffuse–Specular Surface Characteristics, AIAA Paper No. 75–682, AIAA 10th Thermophysics Conference, Denver, Colorado, May 27–29, 1975.

43. Howell, J. R., and Siegel, R., *Thermal Radiation Heat Transfer,* Vol II, NASA SP–164, 1969, Washington, D.C., pp. 70–107.

44. Zienkiewicz, O.C., *The Finite Element Method in Engineering Science,* McGraw–Hill, London, 1971, pp. 116–120.

NASA-Langley, 1977